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ADAPTIVE LOGISTICS SUPPORT
FOR
COMBAT

by

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Thesis Advisor: Donald P. Gaver

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Adaptive Logistics Support
for
Combat

by

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ABSTRACT

The transient behavior of combat logistics support systems is analyzed. Combat availability is defined as the number of active combatant platforms being supported by a single fault diagnosis and repair facility. Heavy traffic conditions inherent to intense combat periods allow the use of diffusion approximation models, which provide speedy solutions used to compare adaptive scheduling policies to a standard First-Come, First-Serve policy. The adequacy of these models is investigated and numerical solutions are compared to simulation results. The case in which failed modules require a degree of support that is beyond the capability of local maintenance is also investigated for both pre- and post-local-repair relocation to distant repair. The use of cannibalization in short-term situations is shown to have a dramatic effect in terms of combat availability. A preliminary model for a non-cannibalization policy is also developed. Optimization models for choosing spare parts allocation within budget constraints, or for achieving required operational availability with minimum cost are described.

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I. INTRODUCTION

A. THE NATURE OF THE PROBLEM

At important times in military operations, especially during intense conventional combat periods, the problem of maintaining the readiness and operational availability of combatant units is of great relevance. Usually the available repair facility is restricted in terms of diagnosis and/or maintenance capability, since it also constitutes part of the deployed group. Thus a detachment unit is expected to operate as successfully as possible for a certain period of high activity, during which relatively many equipment failure events are likely to occur. The purpose of this thesis is to show how the desired availability can be enhanced.

As an example, one can imagine a Carrier-based Air Unit the aircraft of which experience diverse failures of different mission-essential modules; the failures can be either total or partial. A total failure indicates that the original parts inventory may be depleted permanently with the removal of these components from service, while partial depletion signifies that these failed modules are repairable: they can either be immediately served or else join a queue for future service. Also, there is the case in which a failed component arriving at the initial diagnosis station, even if not rendered completely useless, is indicated to require a degree of support that is beyond the capability of local maintenance ("BCM"). The alternative may be to send this failed module to other maintenance levels, with consequent transit and repair times uncertainty being introduced. Because of the delays involved it is clearly important to identify as BCM only those failed components that truly need the distant service.

A basic and important operational problem can be summarized as follows: how to determine a good schedule for repairing failed modules in order to maximize combat availability, subject to various resource restrictions? Usually there is no fundamental reason for servicing failed units in the order in which they fail ("First-come,First-serve" policy), although it is natural and superficially "democratic" or "fair" to do so. Other service policies, allowing for queue length influence, may very well grant increased system availability as a function of time, defined as the total number of operational units "up" at any time t.

This thesis develops and exercises various mathematical models for evaluating scheduling and spares stockage rules in a transient dynamic combat environment. It is very important that adaptive combat logistics support models have the ability to produce a description of transient behavior, since a steady-state situation may never be achieved. It is also desirable to get a solution for both the mean number of components in the system at each time t and the variances as well, since this knowledge will permit the use of various measures of effectiveness which are closer in meaning to standard definitions of availability, i.e., the probability that the number of available units, e.g. carrier-based aircraft, exceed a pre-specified value at each time t.

In this study operational units will be referred to as aircraft, and modules may be considered as being major avionics components, distinct in each aircraft, but assumed to be essential to the operation. It is important not to let the generality of the problem be obscured by these considerations.

Stochastic models for these distributed logistic systems have been derived and verified via simulation for situations such as one server at the repair facility, and Markovian failure and repair times. These analytical models may be employed as tools to analyze the effect of different service disciplines, instead of the exclusive use of time-consuming and large computer-intensive simulation techniques, such as DYNAMETRIC, a package used at RAND Corporation. It is emphasized that the development of the present analytical modeling methodology is in its infancy, and that DYNAMETRIC remains a standard valuable tool.

B. DEFINITIONS

Mathematical definitions and formulations follow.

Let the index i identify each of the different types of components to be considered; $i = 1,\ldots,I$. The Aircraft Unit, together with its base of operation, deploys with K_i components of type i, considering components effectively installed in the aircraft plus any spares to be kept on base, e.g. in the Aircraft-Carrier local parts inventories, to replace those completely lost through attrition or to keep the aircraft operational while failed components undergo repair or wait for service. K_i may actually change over the period of combat as some items permanently fail but others are added.

A local maintenance shop is available to provide service/repair for failed components. It is assumed that components have independent exponential failures at constant rate λ_i and that times to repair are also independent exponential with rate ν_i; our methods will actually accommodate time-dependent λ_i and ν_i. The single repairman at the repair shop "sees" arrival rates of failed components which are equal to individual failure rates multiplied by the total number of items operating at each particular time. This number is, obviously, the number of aircraft actually operational at that time. At any time, t let $N_i(t)$ be the number of components of type i waiting in queue or being serviced.

Let A_c be the number of aircraft initially deployed, and let $A_V(t)$ denote the number of operational (i.e., available) aircraft at time t. $A_V(t)$ is defined by

$$A_V(t) = \min \{A_c, K_1 - N_1(t), K_2 - N_2(t), \ldots, K_I - N_I(t)\} \ . \tag{1.1}$$

As defined earlier, the time-dependent failure rate of each type of component, as seen by the repairman, is equal to $\lambda_i A_V(t)$, for $i=1,2,\ldots,I$. At each time, t there are K_i - $N_i(t)$ components of type i available for use. The definition of $A_V(t)$ in (1.1) follows from the fact that the least available item determines the total availability of operational aircraft, if less than A_c. Note that in this formulation A_c can be a function of time if attrition due to combat is considered.

Equation (1.1) assumes that only one item/module of each type is installed in each of the A_c aircraft, and that **cannibalization** is allowed, i.e., an inactive (due to failure in component type j) aircraft may be removed from its working components in order to permit another unit, which suffered a failure of a different component, to return to active status.

Another assumption that will be needed later is that the system is in *heavy traffic*, i.e.,

$$\min \{A_c, K_1, K_2, \ldots, K_I\} \times \sum_{i=1}^{I} \frac{\lambda_i}{\nu_i} > 1 . \tag{1.2}$$

This assumption appears to be very realistic for most deployments, specially in a real combat situation.

Considering now the repairman side of the problem, it seems clear that it is costly, in terms of time, to switch from the component being currently repaired to a new one before service termination and, consequently, the service strategies to be considered will determine the next component to be serviced (to completion) at the moment of the previous service completion. Of course if a server incumbent is not finished for an extraordinarily long time it may well be desirable to interrupt its service and substitute another now more vital item. Rules for such a substitution are not evaluated in this thesis. It is clear that under some circumstances such procedures can be useful adaptations.

C. GOALS AND SCOPE

The present work attempts to exploit stochastic models developed for similar systems, as well as to perform initial sensitivity analysis with respect to certain tuning parameters.

Monte Carlo simulations are employed as validation tools, and particularly extreme numerical examples are investigated so as to build understanding of the capabilities of this approach.

Also, as mentioned earlier, the original models are augmented in order to be able to consider the possibility of module failures which are "beyond the capability of maintenance". The objective is to show that, with small modifications, these models are capable of providing adequate approximations for the somewhat more realistic "BCM" problem.

The effect of not using the cannibalization policy is investigated via a simulation model, and a modification to the analytic model is proposed to account for this change in strategy.

Finally, an optimization model is proposed, allowing for considerations of limited resources and for the need of quick allocation of such resources to spares for the various modules, accounting for the effect of following an adaptive repair scheduling policy, so as to maximize overall system availability.

II. REVIEW OF EXISTING MODELS - BACKGROUND

Models have been developed to analyze the transient behavior of similar systems with respect to the selection of a particular service policy. Initially, a natural choice for a Measure of Effectiveness ("**MOE**") is given by the expected number of aircraft available at time t, $E[A_V(t)]$, (1.1) or, nearly equivalently, by the set of expected values of available modules of each type, $E[K_i - N_i(t)]$, $i=1,2,\ldots,I$.

A. SIMULATION

Several maintenance policies have been compared in a previous work by Latta [Ref. 1]. Simulation was employed to study six different repair policies going from the original First-Come, First-Serve ("**FC,FS**") scheme to what can be called *Least Available Item Next* ("**LAIN**") policy. This last scheduling strategy determines service priority based on the current availability of all types of components, and in which the repairman, after scanning inventory levels (if any), re-orders components in the service queue to favor those with the lowest operational stock level. If all initial stocks are completely depleted at any time t, the component type with lowest value of $K_i - N_i(t)$ is chosen to be serviced next.

Latta demonstrated that the **LAIN** policy yields considerable improvement in terms of mean number of available aircraft over all other considered policies.

A clear disadvantage of the simulation technique for exploring our model implications is the intensive requirement for computer usage inherent in the straightforward Monte-Carlo simulation approach. It seems prohibitively costly, if not entirely impractical, to employ these models in a optimization context, for example, or in cases where imprecisely determined parameters for failure and repair times indicate the need for some kind of dynamic assessment of uncertainty in these parameters, such as by "bootstrapping". Real problems involve aspects of both of the issues; this paper has something to say about each.

B. STOCHASTIC MODELS

1. Definitions

Diffusion processes are particular cases of Gaussian processes with continuous sample functions, and were originally used to model physical phenomena, e.g. the motion of small particles of gas. The Brownian Motion (or Wiener process) $\{W_t , t \geq 0\}$ is the most elementary example of a diffusion process, having zero drift (or infinitesimal mean), and diffusion parameter (or infinitesimal variance) independent of W. It is the continuous time version of a random walk.

A diffusion process $\{X_t , t \geq 0\}$ satisfying the stochastic differential equation

$$dX_t = -\rho X_t + \sigma dW_t \ , \quad X_0 = c \ ; \ \rho , \sigma > 0 \qquad (2.1)$$

is called an Ornstein-Uhlenbeck process. This stochastic process was first used to directly model the velocity of a particle subject to elastic forces. In this case, the drift parameter reflected a restoring force proportional to the distance, and directed towards the origin.

For a complete treatment of diffusion processes see, for example, Karlin and Taylor [Ref. 2]. Arnold [Ref. 3] examines the multivariate stochastic differential equations for the Ornstein-Uhlenbeck process, and gives general results for the mean and covariance of X_t in (2.1).

2. Air Unit Detachment Model and Related Work

Several papers have discussed the use of diffusion processes as an approximation for job-server systems under heavy traffic. See, for example, Gaver and Jacobs [Ref. 4], Gaver and Lehoczky [Ref. 5 and 6] and Iglehart [Ref. 7].

Gaver and Jacobs [Ref. 4] developed diffusion approximation models for computer systems with processor-shared service disciplines. Gaver and Lehoczky [Ref. 6] studied a repairman problem with two types of repair. Pilnick [Ref. 8] extended these models to account for multiple types of queues and service priority proportional to a function of queue length and modeled the Air Unit detachment problem directly. Gaver, Isaacson and Pilnick [Ref. 9] exploit these models and presented various applications. The results are summarized here.

a. *Processor-sharing Adaptation*

If we define $q_i(\mathrm{N}(t))$, where $\mathrm{N}(t)$ is the vector with components $N_i(t)$, as the proportion of time jobs of type i are served by the processor, we can view

$q_i(\mathbf{N}(t))$ as the probability that the processor (server) will select job i for service just after each time slice, when a departure takes place in that time slice of length dt.

In order to derive mathematical models for the actual Air Unit problem, $q_i(\mathbf{N}(t))$ is used to represent the probability that, after service completion at time t, a module of type i is selected for being serviced next. In this case, it turns out that $q_i(\mathbf{N}(t))$ is, in fact, a function of $\mathbf{N}(t)$ and $A_V(t)$, i.e., it depends on the queue sizes and the operational (combat) availability. In fact, $q_i(\mathbf{N}(t))$ is a decision variable, subject to determination by the scheduler. For notational simplicity, denote $q_i(\mathbf{N}(t))$ by $q_i(t)$.

b. Diffusion Approximation - The Air Unit Detachment Problem

Pilnick [Ref. 8:p.143] showed how the following system of stochastic differential equations can be obtained:

$$dN_i(t) = \lambda_i A_V(t)\,dt - \nu_i p_i(t)\,dt + \sqrt{\lambda_i A_V(t) + \nu_i p_i(t)(1 + 2p_i(t)\{\nu_i \sum_j \frac{p_i(t)}{\nu_j} - 1\})}\; dW_i(t) \qquad (2.2)$$

where $\{W_i(t), t \geq 0\}$ are independent standard Wiener processes, $N_i(t)$, $i=1,2,\ldots,I$ are continuous approximations of the actual discrete processes, and

$$p_i(t) = C \frac{w_i}{\mu_i} q_i(t) \quad ,$$

where C is a normalization constant depending on the specific form of $p_i(t)$, and $w_i > 0$ denote arbitrary scalars representing weights for items of type i; see Chapter III-A below .

Assume that the system is large, for example let $a=A_c+\Sigma_i K_i$, $A_c=a.a_c$, $K_i=a.\alpha_i$, and $\mu_i=a.\nu_i$ are constants. Define $\beta_i(t)=N_i(t)/a$, where $\beta_i(t)$ approaches a deterministic function of t representing the scaled mean of $N_i(t)$, as $a\rightarrow\infty$.

Consider the normalization/approximation

$$X_i(t) \sim \frac{N_i(t) - a\,\beta_i(t)}{\sqrt{a}} \quad . \tag{2.3}$$

(1) Expected Values. If we define $dN_i(t)$ as the increment $N_i(t+dt)$-$N_i(t)$ and express it in terms of transformations similar to those in (2.3), it can be shown (see Pilnick [Ref. 8]) that the following system of I ordinary differential equations can be derived for $\beta_i(t)$, $i=1,2,...,I$:

$$\frac{d\beta_i(t)}{dt} = \lambda_i\, a_V(t) - \mu_i\, p_i(t) \tag{2.4}$$

where $a_V(t)$ is a scaled version of $A_V(t)$ in (1.1), i.e.

$$a_V(t) = \min\{a_c, \alpha_1 - \beta_1(t), \ldots, \alpha_I - \beta_I(t)\} \tag{2.5}$$

and $q_i(t)$ are smooth functions chosen to be a representation of service policy. Different definitions for $q_i(t)$ will be described in Chapter III.

(2) Variances. A scaled variance-covariance matrix of $N_i(t)$, $\Sigma(t)$ can be also defined through a system of $I\times I$ ODE's. Following the notation in Pilnick [Ref. 8, pp. 85,147], let

$$\Sigma(t) = \begin{bmatrix} \sigma_{11}(t) & \sigma_{12}(t) & \dots & \sigma_{1I}(t) \\ \sigma_{21}(t) & \sigma_{22}(t) & \dots & \sigma_{2I}(t) \\ \vdots & \vdots & \dots & \vdots \\ \sigma_{I1}(t) & \sigma_{I2}(t) & \dots & \sigma_{II}(t) \end{bmatrix}$$

be the variance-covariance matrix of $\mathbf{N}(t)$ scaled by (2.3). The equations for the elements of $\Sigma(t)$ are

$$\frac{d\sigma_{ii}(t)}{dt} = B^2{}_{ii}(t) + 2\sum_{j=1}^{I} H_{ij}\,\sigma_{ij}(t)\ ; \tag{2.6}$$

and, for $i \neq j$,

$$\frac{d\sigma_{ij}(t)}{dt} = \sum_{k=1}^{I} \left[H_{ik}(t)\,\sigma_{jk}(t) + H_{jk}(t)\,\sigma_{ik}(t) \right]\ . \tag{2.7}$$

Here, $\mathbf{H}(t)$ is the $I \times I$ matrix

$$H_{ij}(t) = \begin{cases} H^1_{ij}(t) & ,\ if\ j \neq s(t) \\ H^1_{ij}(t) - \lambda_i, & if\ j = s(t) \end{cases} ,$$

$s(t) = argmin\ \{\ \alpha_1 - \beta_i(t), \dots, \alpha_I - \beta_I(t)\ \}$, with

$$H^1_{ii}(t) = -\ \mu_i \frac{\rho\,\gamma}{\xi_i + \rho\,\beta_i(t)}\, p_i(t)(1 - p_i(t))\ \ , \tag{2.8}$$

and, for $i \neq j$,

$$H^1_{ij}(t) = \mu_i \frac{\rho\,\gamma}{\xi_j + \rho\,\beta_j(t)} p_i(t) p_j(t) \quad . \tag{2.9}$$

The scalars ξ_i , ρ and γ are determined by the choice of the service policy, as we will see in Chapter III.

$\mathbf{B}^2(t)$ in (2.6) is the $I \times I$ diagonal matrix with elements

$$B^2_{ii}(t) = \lambda_i a_V(t) + \mu_i p_i(t) \left(1 + 2 p_i(t) \{ \mu_i \sum_j \frac{p_j(t)}{\mu_j} - 1 \} \right) .$$

III. ADAPTIVE SUPPORT LOGISTICS

Chapter II introduced the general analytic expressions for the mean and the variance-covariance function of an approximation to $\mathbf{N}(t)$, valid for a large system ("large" suitably defined) in heavy traffic. In this chapter, adaptive scheduling policies are studied, and compared to a standard **FCFS** policy. The accuracy of the diffusion approximation is verified in various numerical examples by comparison with results of Monte Carlo simulations. The behavior of the analytic models with respect to certain tuning parameters is demonstrated in cases showing wide variation of repair and failure rates. Under suitable conditions the analytical approximation is in excellent agreement with simulation, and is conducted in a fraction of the computer time required by present simulations; see below.

Most of the numerical solutions of the differential equations were obtained on an IBM 3033/4381 computer at the Naval Postgraduate School using the IMSL (Release 10) package's ODE solvers with either Adam's method or Gear's stiff method (backward differentiation formula). All codes were written in such a way that immediate translation to PC-FORTRAN is possible.

All simulations were carried out on the IBM 3033/4381 mainframe, using the LLRANDOMII random number generator package [Ref. 10]. Certain relevant details of the simulations are provided when necessary in the next sections.

It should be mentioned that, for 1000 replications, the time spent in the simulation models was, on average, on the order of eight minutes, while the numerical solutions of the differential equations usually took approximately one second for the examples under study.

A. MODELING ADAPTIVE POLICIES

As was mentioned in Chapter II, in the diffusion approximation model is essential to characterize the availability and queueing behavior induced by a scheduling policy represented by the choice of $q_i(t)$. Pilnick [Ref. 8] describes fairly general expressions for $q_i(t)$ that can be useful for modeling several service policies. Put simply, let

$$p_i(t) = \frac{w_i q_i(t)/\mu_i}{\sum_j w_j q_j(t)/\mu_j} \tag{3.1}$$

where $q_i(t)$ is the "probability" (a relative measure) that an item of type i is selected for service next, if the previous service is completed at t. The following fairly general form of $q_i(t)$ incorporates a plausible definition for the parameters ρ, γ and ξ_i in (2.8) and (2.9):

$$q_i(t) = \frac{(\xi_i + \rho\,\beta_i(t))^\gamma}{\sum_j (\xi_j + \rho\,\beta_j(t))^\gamma} \quad . \tag{3.2}$$

The actual numerical values assigned to the parameters will depend upon the selection of the scheduling policy. For our present purpose, two forms for $q_i(t)$ are investigated.

1. **First-Come, First-Served (FC,FS)**

A technique to model this common policy is to suppose that the probability of choosing a module of type i for being serviced next is proportional to $N_i(t)$. In this case we must put $\rho=1$, $\gamma=1$ and $\xi_i=0$ and (3.2) becomes

$$q_i(t) = \frac{\beta_i(t)}{\sum_j \beta_j(t)} \quad , \tag{3.3}$$

2. **Least Available Item Next (LAIN)**

Gaver, Isaacson and Pilnick [Ref. 9] devised the term "anti-availability" of a module i to characterize the quantity $(K_i\text{-}N_i(t))^{-1}$. If $q_i(t)$ is defined as

$$q_i(t) = \frac{[\alpha_i - \beta_i(t)]^{-p}}{\sum_j [\alpha_j - \beta_j(t)]^{-p}} \quad , \tag{3.4}$$

where p is a large integer (e.g., 10 - 40), then (3.4) is an attempt to emulate a deterministic choice of the module with largest anti-availability. This present form of $q_i(t)$ implies that we must have $c=-1$, $\gamma=-p$ and $\xi_i=\alpha_i$ in (3.2).

a. *Sensitivity Analysis*

It turns out that the selection of the tuning parameter p in (3.4) influences the accuracy of the diffusion approximation, depending upon the range of values for failure and repair rates.

In this section two hypothetical cases are examined: the first (Case 1) exhibits comparable values for the rates and it is demonstrated that low values of p yield good precision; the second case (Case 2) illustrates the fact that a larger value of p must be used when the rates vary considerably. For both cases in this section, $A_c = 50$ and $I = 10$, and no spares are provided. It is assumed that we are interested in evaluating combat availability for missions of duration up to $T=100$ (e.g., days).

Simulation results are used to establish the basis for comparison. Here, as well as in all numerical examples for the LAIN policy in this thesis, the simulation model chooses the actual least-available module for service next as it moves along the sample path in each replication. The analytical method represents a probabilistic selection, but one that with near certainty picks the least available item for repair.

(1) *Case 1.* Table 3.1 displays the input data for the first case. Failure and repair rates are given with respect to a "standard" time unit (e.g., days).

Table 3.1 CASE 1 INPUT DATA

MODULE	K_i	λ_i	ν_i
1	50	0.020	5.0
2	50	0.021	5.0
3	50	0.022	5.0
4	50	0.023	5.0
5	50	0.024	5.0
6	50	0.025	4.5
7	50	0.026	4.5
8	50	0.027	4.5
9	50	0.025	4.5
10	50	0.020	4.5

In Figure 3.1, computed values for $E[A_V(t)]$ are plotted for several values of p. Simulation results are the actual mean values of aircraft availability at each t. Analytical solutions are computed at each time t according to the formula

$$E[A_v(t)] = \min\{A_c, K_1 - a.\beta_1(t), \ldots, K_I - a.\beta_I(t)\}. \tag{3.5}$$

Formula (3.5) is correct to order a only; in practical terms it does not recognize the random variability of K_i-$N_i(t)$. Note that, at all t values examined, there is fairly good agreement between simulation and analytical solutions, even for small values of p.

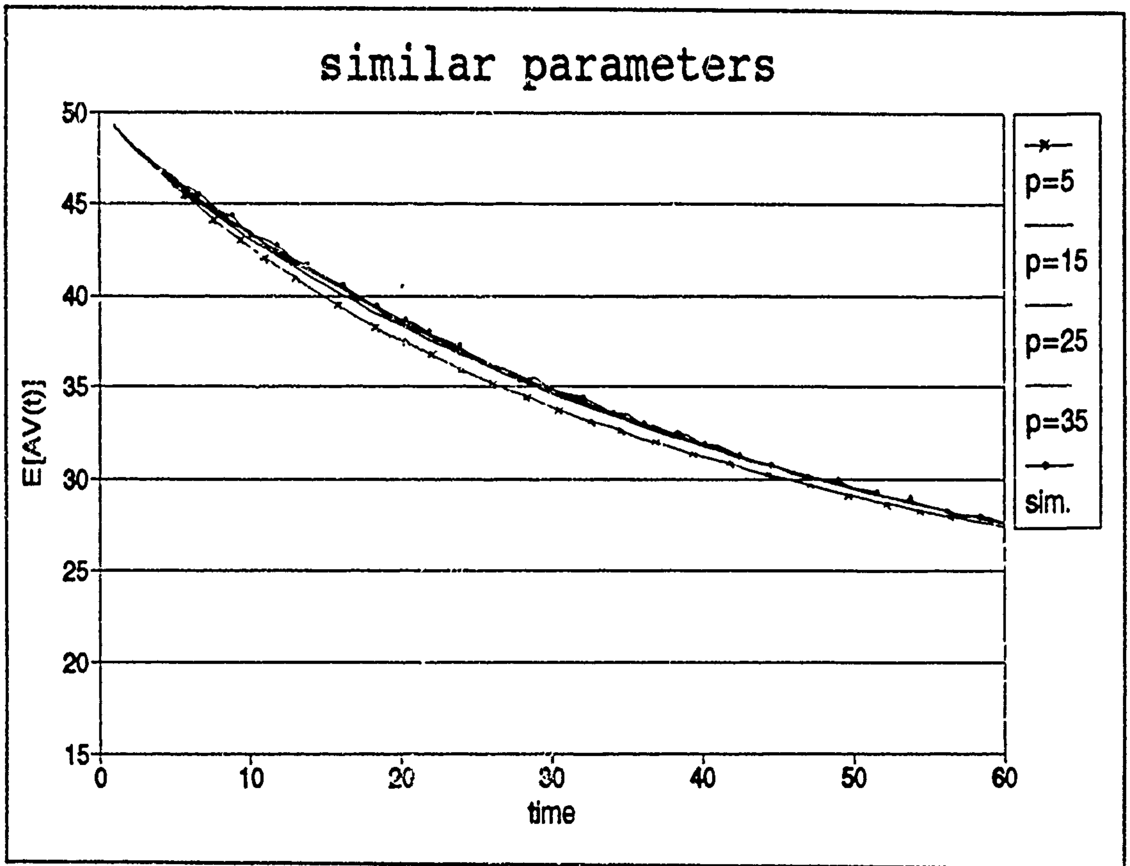

Figure 3.1 Case 1 Availability

(2) Case 2. Table 3.2 shows that failure and repair rates for Case 2 present much higher variation. As in Case 1, the sample system comprises $A_c=50$ aircraft having each $I=10$ vital modules. All aircraft have exactly one module of each type installed.

Table 3.2 CASE 2 INPUT DATA

MODULE	K_i	λ_i	ν_i
1	50	0.005	5.0
2	50	0.005	5.0
3	50	0.005	5.0
4	50	0.005	5.0
5	50	0.005	5.0
6	50	0.009	2.5
7	50	0.011	2.5
8	50	0.013	2.5
9	50	0.015	2.5
10	50	0.025	2.5

As Figure 3.2 depicts, the choice of the parameter p now more strongly influences the accuracy of the analytical solutions. Note that $p \geq 30$ is required to obtain a good approximation in this case.

The importance of these initial results is that the user of the present analytical models should be aware that the selection of the appropriate tuning parameter is not case-independent. A reasonable implementation of these analytical models should consider a pre-verification of rate ranges before assigning the value of p. For technical reasons, however, it is desirable to make p as small as possible.

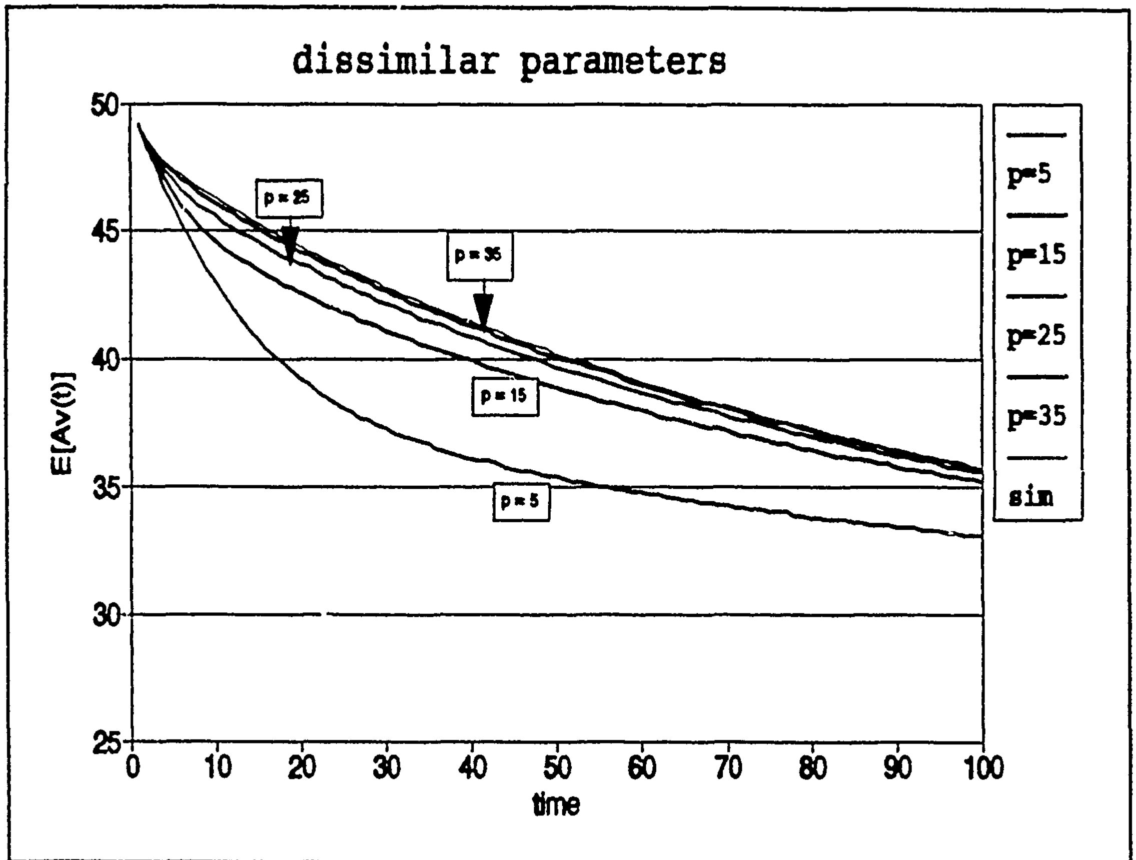

Figure 3.2 Case 2 Availability

B. NORMALITY ANALYSIS

An important result involving the diffusion approximation for large systems under heavy traffic conditions (1.2) is that N(t) in (2.2) is approximately multivariate normal with mean $a\beta(t)$ and variance-covariance matrix $a\Sigma(t)$. Consequently, K-N(t) must be also normal with mean K - $a\beta(t)$ and the same variance. Here $\beta(t)$ is the vector with components $\beta_i(t)$, $i=1,\ldots,I$.

Pilnick [Ref. 8, p.100-102] applied classical statistical analysis to the simulation data and demonstrated that the normality assumption appears to hold for particular transient times, as well as for the steady-state phase. In this section, alternative non-parametric methods are employed to verify the normality theory for the complete mission period.

1. Methodology

If the hypothesis of normality holds for $A_V(t)$, then the 5^{th} and 95^{th} percentiles of the distribution of $A_V(t)$ are approximately

$$\begin{aligned} \varphi_{.05} &= E[A_V(t)] + z_{.05} \times \sqrt{Var[A_V(t)]} \\ \varphi_{.95} &= E[A_V(t)] + z_{.95} \times \sqrt{Var[A_V(t)]} \quad , \end{aligned} \tag{3.6}$$

where z_α is the α^{th} quantile of a standard normal distribution.

Values of $E[A_V(t)]$ and $Var[A_V(t)]$ can be computed by simulation and, importantly, also by an analytical-numerical method, and (3.6) can be used to calculate $\varphi_{.05}$ and $\varphi_{.95}$. Now suppose samples of size $n=1000$ (i.e., 1000 replications in the simulation) are generated. If the 50^{th} and 950^{th} sample order statistics are stored for each time t, then, under the normal hypothesis, it is presumed that these values will approximate the theoretical numbers.

2. Numerical Results

The sample system represented by Case 2 above is used as a numerical example. Figure 3.3 displays the mean value of $A_V(t)$, the theoretical percentiles computed by (3.6), and the corresponding sample order statistics. Note the jumps in the order statistics, denoted by A(50) and A(950) in the plot. This is explained by the fact that they are integer values, representing actual aircraft availability at t for a particular replication. It is clear that the agreement is very satisfactory, confirming the usefulness of the normal model approximation. It thus becomes attractive to compute the probability that the number of available aircraft at time t is less than or equal to x by use of a normal approximation.

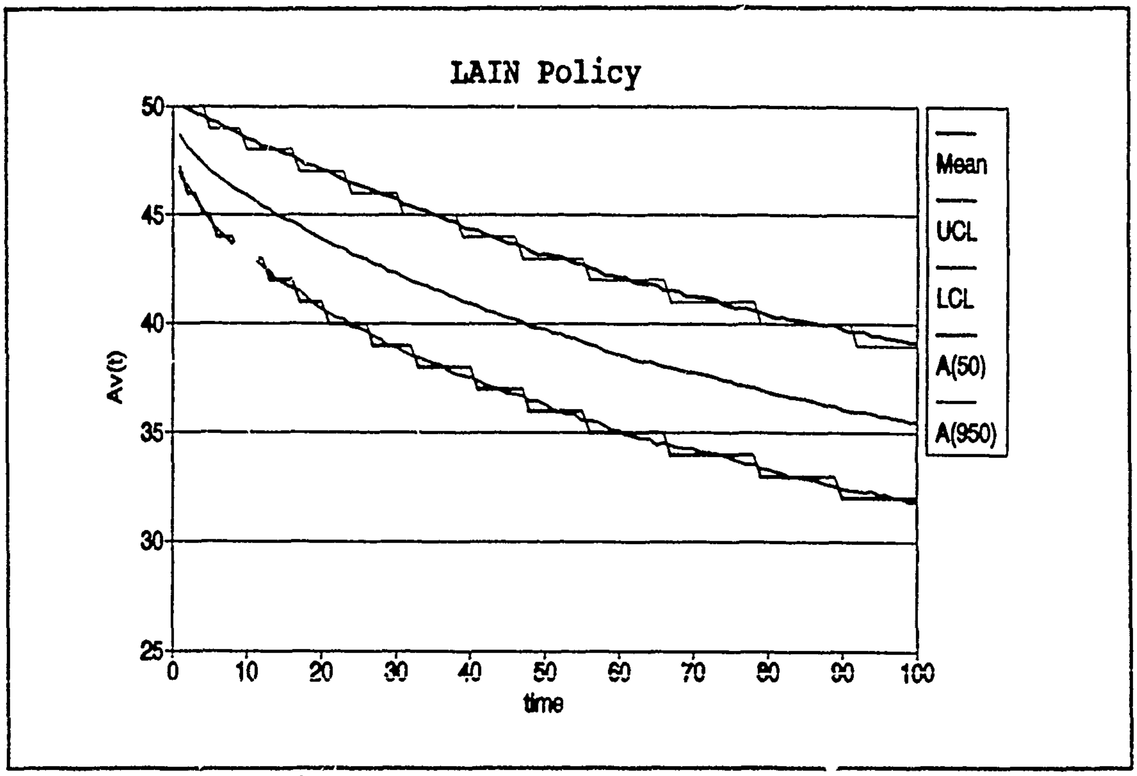

Figure 3.3 Test for Normality

C. CASE STUDY

In Table 3.3 input data for a particular system are presented. This example is taken from Gaver, Isaacson and Pilnick [Ref. 9], who analyzed the effect of different stockage patterns using the analytical formulation. Once more, failure and repair rates show large variation. The objective in this section is to reproduce the analysis of the relative performances of the adaptive (LAIN) and FCFS policies, using simulation and analytical techniques.

Table 3.3 CASE STUDY INPUT DATA

MODULE	K_i	λ_i	ν_i
1	50	0.050	5.0
2	50	0.040	5.0
3	50	0.030	5.0
4	50	0.020	5.0
5	50	0.010	5.0
6	50	0.009	2.5
7	50	0.008	2.5
8	50	0.007	2.5
9	50	0.006	2.5
10	50	0.005	2.5

Several modifications to the original set-up are introduced and the consequent effect on aircraft availability is analyzed. For all cases, simulation (S) and analytical (A) results are tabulated for both policies. These results are the mean values (E[AV]), standard deviation (SDEV), and the 5th and 95th percentiles (95% CI) of the distribution of $A_V(t)$. Appendix A reproduces these results in graphic form; for each case that is analyzed in this section, plots are provided for comparative performance for the two policies, as well as probabilistic limits for the actual availability using both simulation and analytical formulations.

In this sample case there are initially A_c = 50 aircraft. For the LAIN policy, the "anti-availability" parameter p is set to 30 in (3.3). Table 3.4 exhibits the resulting combat availability for each policy when no spares are provided, i.e., K_i=50, i=1,2,...,10 (Case A).

Table 3.4 COMPARATIVE PERFORMANCES (CASE A)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	38.57	2.97	39.27	3.31	43.38	2.10	43.18	2.55
20	32.32	3.28	32.76	3.78	40.22	2.06	40.31	2.51
30	28.66	3.63	28.81	3.89	38.01	2.13	38.13	2.40
40	26.43	3.58	26.42	3.90	35.92	2.13	36.30	2.30
50	25.19	3.65	24.95	3.90	34.36	2.10	34.71	2.21
60	24.06	3.43	24.08	3.89	33.03	1.96	33.32	2.14
70	23.47	3.60	23.54	3.89	31.81	1.85	32.10	2.07
80	22.93	3.70	23.22	3.89	30.90	1.86	31.03	2.02
90	23.00	3.71	23.02	3.89	29.90	1.92	30.08	1.97
100	22.50	3.67	22.91	3.89	29.09	1.89	29.23	1.92
	95%	CI	95%	CI	95%	CI	95%	CI
10	43.46	33.68	44.72	33.82	46.85	39.91	47.39	38.97
20	37.73	26.91	38.99	26.53	43.62	36.83	44.45	36.18
30	34.65	22.67	35.23	22.39	41.52	34.49	42.09	34.17
40	32.34	20.51	32.86	19.98	39.44	32.41	40.09	32.51
50	31.21	19.16	31.38	18.52	37.83	30.89	38.36	31.06
60	29.72	18.39	30.50	17.65	36.26	29.81	36.85	29.79
70	29.40	17.54	29.96	17.13	34.86	28.77	35.52	28.68
80	29.03	16.82	29.63	16.81	33.96	27.83	34.35	27.70
90	29.12	16.87	29.44	16.61	33.06	26.73	33.32	26.83
100	28.55	16.45	29.32	16.49	32.04	26.13	32.40	26.06

As it was expected, the LAIN priority policy achieves larger average values for combat availability at all times. Note, also, that the adaptive scheme induces lower variances than does the simple FCFS policy. It should be noted especially that the diffusion approximation results match those from the simulation quite accurately. At least for this example the standard deviations derived using analytical method (A) are slightly larger than those from simulation (S). If this slightly conservative bias prevails it is a desirable situation.

In Case B one attempts to increase combat availability by providing two spares for each module. Table 3.5 displays the results for Case B.

Table 3.5 COMPARATIVE PERFORMANCES (CASE B; 2 SPARES)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	39.89	3.17	40.53	3.39	45.03	1.96	44.65	2.65
20	32.95	3.67	33.52	3.86	41.48	2.02	41.51	2.60
30	28.88	3.81	29.28	3.97	38.96	2.23	39.17	2.49
40	26.46	3.77	26.71	3.97	36.75	2.18	37.22	2.38
50	24.99	3.84	25.14	3.95	35.14	2.17	35.53	2.28
60	24.13	3.56	24.19	3.94	33.59	2.06	34.06	2.20
70	23.58	4.08	23.62	3.94	32.41	2.00	32.76	2.12
80	23.25	3.99	23.27	3.93	31.23	1.93	31.62	2.06
90	23.28	3.86	23.05	3.93	30.37	1.89	30.62	2.01
100	23.18	3.85	22.92	3.93	29.55	1.92	29.73	1.96
	95% CI		95% CI		95% CI		95% CI	
10	45.11	34.66	46.11	34.94	48.26	41.80	49.02	40.29
20	39.00	26.89	39.89	27.16	44.81	38.16	45.81	37.22
30	35.16	22.60	35.82	22.73	42.64	35.29	43.29	35.06
40	32.67	20.24	33.26	20.15	40.35	33.16	41.14	33.30
50	31.32	18.66	31.66	18.62	38.73	31.56	39.29	31.77
60	30.00	18.26	30.69	17.70	36.99	30.20	37.68	30.43
70	30.31	16.84	30.11	17.12	35.71	29.10	36.27	29.26
80	29.84	16.67	29.75	16.78	34.41	28.05	35.03	28.22
90	29.64	16.92	29.54	16.56	33.49	27.24	33.93	27.31
100	29.47	16.90	29.41	16.43	32.71	26.39	32.96	26.49

The numbers show a small improvement in the average availability, specially in early periods; however, there is very small improvement at later times.

The effect of providing ten spare parts for each module (Case C) is examined next; see Table 3.6.

Table 3.6 COMPARATIVE PERFORMANCES (CASE C; 10 SPARES)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	45.56	3.23	46.40	3.83	49.53	1.80	50.00	3.42
20	36.84	3.97	37.17	4.19	46.21	2.69	46.61	3.26
30	31.17	4.01	31.46	4.24	43.27	2.80	43.30	3.08
40	27.93	3.93	28.01	4.20	40.62	2.65	40.83	2.87
50	26.03	3.90	25.93	4.15	38.47	2.54	38.80	2.66
60	24.74	4.07	24.66	4.12	36.65	2.48	37.03	2.51
70	23.67	3.97	23.89	4.10	35.16	2.36	35.45	2.40
80	23.42	4.02	23.43	4.09	33.78	2.34	34.06	2.31
90	23.06	3.77	23.15	4.08	32.62	2.22	32.83	2.22
100	23.01	3.83	22.98	4.08	31.68	2.10	31.74	2.16
	95% CI		95% CI		95% CI		95% CI	
10	50.89	40.24	52.91	40.28	51.50	45.55	55.65	44.35
20	43.39	30.28	44.09	30.26	50.66	41.77	51.99	41.22
30	37.67	24.46	38.46	24.46	47.77	38.73	48.38	.21
40	34.41	21.45	34.93	21.09	44.96	36.23	45.56	36.09
50	32.57	20.02	32.77	19.08	42.56	34.38	43.19	34.42
60	31.47	18.34	31.45	17.87	40.70	32.50	41.17	32.88
70	30.04	17.30	30.65	17.13	38.98	31.20	39.41	31.49
80	29.96	16.75	30.17	16.68	37.65	29.91	37.86	30.25
90	29.28	16.84	29.88	16.41	36.28	.95	36.50	29.17
100	29.34	16.68	29.71	16.25	35.14	28.22	35.30	28.19

A significantly greater effect on aircraft availability occurs in Case C. The improvement in availability is specially noticeable in early times for both policies, with the effect decreasing considerably at the end of the mission period. Note, also, that the effect is more important in the LAIN availability. In the LAIN case, there is a much larger relative error between the diffusion approximation and simulation with respect to standard deviations for early times; again the standard deviation obtained analytically (LAIN-A) noticeably exceeds the simulation value (LAIN-S).

It is interesting to observe what happens when ten spares are allocated only to those five modules with higher failure rates (Case D); see Table 3.7.

Table 3.7 COMPARATIVE PERFORMANCES (CASE D)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	46.95	2.44	46.99	3.98	46.17	1.83	46.11	2.90
20	36.74	3.78	37.43	4.21	42.72	2.07	42.59	2.74
30	31.32	4.27	31.62	4.24	39.80	2.19	39.97	2.63
40	28.05	3.90	28.10	4.20	37.55	2.23	37.95	2.50
50	26.12	3.67	25.98	4.15	35.62	2.30	36.22	2.36
60	24.74	4.01	24.69	4.12	34.14	2.07	34.64	2.24
70	23.81	3.95	23.90	4.10	32.91	1.99	33.24	2.17
80	23.55	3.99	23.44	4.08	31.87	1.93	32.01	2.09
90	23.49	3.84	23.16	4.08	30.79	1.91	30.94	2.03
100	23.23	3.83	22.98	4.08	29.84	2.00	29.99	1.98
	95% CI		95% CI		95% CI		95% CI	
10	48.97	40.92	53.56	40.43	49.18	43.16	50.89	41.33
20	42.98	30.50	44.37	30.49	46.13	39.31	47.11	38.07
30	38.36	24.28	38.61	24.62	43.42	36.18	44.31	35.62
40	34.48	21.62	35.03	21.18	41.24	33.87	42.08	33.82
50	32.17	20.07	32.83	19.13	39.41	31.84	40.11	32.32
60	31.35	18.12	31.48	17.90	37.55	30.73	38.34	30.95
70	30.33	17.30	30.66	17.14	36.19	29.62	36.80	29.67
80	30.13	16.96	30.18	16.70	35.06	28.69	35.46	28.56
90	29.83	17.16	29.89	16.42	33.95	27.64	34.29	27.59
100	29.55	16.90	29.71	16.26	33.14	26.53	33.26	.73

In this case, the FCFS policy performs slightly better than the priority scheme in the beginning of the mission. The implication may be that an appropriate weighting, taking into account high failure rates, may prove to be more efficient than the exclusive concern with the current least available module at each time. The LAIN policy is again more efficient for later periods.

Now suppose that all modules cost the same (not very realistic, of course), and instead of allocating ten spares to those modules with higher failure rates, as in Case D, five spares are introduced across the board (Case E); see Table 3.8.

Table 3.8 COMPARATIVE PERFORMANCES (CASE E; 5 SPARES)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	42.00	3.22	42.59	3.51	46.95	2.70	46.95	2.83
20	34.14	3.70	34.75	3.97	43.33	2.50	43.33	2.78
30	29.63	3.85	30.00	4.06	40.47	2.40	40.60	2.66
40	27.03	3.80	27.12	4.04	38.30	2.29	38.58	2.51
50	25.29	3.81	25.38	4.03	36.32	2.20	36.71	2.39
60	24.27	3.86	24.33	4.01	34.92	2.21	35.16	2.29
70	23.68	3.87	23.69	4.00	33.44	2.10	33.70	2.21
80	23.43	4.03	23.32	3.99	32.30	2.03	32.51	2.14
90	23.27	3.79	23.09	3.99	31.19	1.98	31.42	2.08
100	22.99	3.66	22.96	3.99	30.22	2.02	30.44	2.02
	95% CI		95% CI		95% CI		95% CI	
10	47.63	36.71	48.39	36.79	50.36	43.54	51.63	42.28
20	41.20	28.07	41.32	28.19	47.38	39.26	47.92	38.74
30	35.87	23.38	36.71	23.29	44.43	36.51	45.10	36.30
40	33.11	20.96	33.82	20.43	42.03	34.49	42.74	34.43
50	31.50	19.07	32.04	18.74	39.95	32.69	40.72	32.81
60	30.64	17.89	30.95	17.72	38.46	31.18	38.95	31.37
70	30.07	17.30	30.30	17.10	36.90	29.98	37.41	30.10
80	30.09	16.80	29.91	16.72	35.65	28.95	36.06	28.98
90	29.53	17.02	29.67	16.49	34.45	27.93	34.86	28.00
100	29.03	16.95	29.53	16.35	33.55	26.89	33.81	27.13

It is illustrative to observe that this new, and almost certainly less expensive, stock plan influences each scheduling policy in a different way. In the FCFS case the combat availability declines throughout the mission when compared to Case D, especially at earlier times; on the other hand, and somewhat surprisingly, LAIN availability is slightly enhanced at all times.

Table 3.9 shows the result of submitting the system in Case E to an arbitrary weighting consisting of $w_j = \nu_j/\lambda_j$ i.e., a high weight for modules with high repair rate and low failure rate, using the LAIN policy.

Table 3.9 THE EFFECT OF WEIGHTS IN CASE E

time	WEIGHTS NOT USED	WEIGHTS USED
10	46.9	46.8
20	43.3	43.2
30	40.6	40.5
40	38.5	38.3
50	36.7	36.5
60	35.1	35.0
70	33.7	33.5
80	32.5	32.3
90	31.4	31.4
100	30.4	30.3

Quite surprisingly, the effect of such weighting procedure is insignificant (in fact, combat availability decreases slightly). It is necessary to understand that the numbers in Table 3.9 were generated by setting $w_i = 1$ and $w_i = \nu_i/\lambda_i$, $i = 1,2,\ldots,10$, in (3.2). The result is obviously the same if the w_i are set to μ_i/λ_i instead of ν_i/λ_i. It is not clear if this fact implies that this arbitrary weighting is ineffective, or if the model does not truly represent such strategy.

Finally, simulations using various weighting factors were run for the LAIN case, including the procedure in which the module type selected for repair next is that with the least value of the product of the actual availability by the ratio failure/repair rates. All the results showed that the combat availability is reduced, when compared to the unweighted LAIN scheme.

In our last sample system (Case F) the parameters are modified in such a way that the heavy traffic condition (1.2) for the diffusion approximation is barely satisfied; see Table 3.10.

Table 3.10 CASE F; INPUT DATA

MODULE	K_i	λ_i	ν_i
1	50	0.045	7.5
2	50	0.040	7.5
3	50	0.030	7.5
4	50	0.020	7.5
5	50	0.010	7.5
6	50	0.009	4.5
7	50	0.008	4.5
8	50	0.007	4.5
9	50	0.006	4.5
10	50	0.005	4.5

If (1.2) is applied to the data we have

$$50 \times \sum_{i=1}^{10} \frac{\lambda_i}{\nu_i} = 1.355 .$$

Table 3.11 shows the computed expected values and standard deviations for $A_V(t)$ for both the simulation (S) and the diffusion approximation (A).

Table 3.11 COMPARATIVE PERFORMANCES (CASE F)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	43.89	2.81	44.28	2.73	46.58	1.46	46.22	2.80
20	41.24	3.30	41.93	3.34	45.17	1.68	44.97	2.15
30	39.65	3.64	40.21	3.53	44.06	1.77	43.91	2.10
40	38.60	3.80	38.94	3.59	43.09	1.87	43.00	1.85
50	37.97	3.77	38.11	3.75	42.25	1.86	42.21	2.27
60	37.51	3.65	37.96	3.66	41.54	1.96	41.52	2.20
70	37.27	3.65	37.73	3.77	40.94	1.95	40.91	2.52
80	37.07	3.75	37.44	3.92	40.35	1.98	40.39	1.99
90	37.10	3.64	37.25	3.87	39.82	1.98	39.93	2.30
100	37.10	3.70	37.13	4.40	39.42	2.01	39.53	2.25

It can be noticed that the approximation provided by the diffusion model is still very useful, especially for mean values; the (now -ger) error in standard deviation is, again, biased upward. A case where the heavy traffic condition is violated is investigated in Appendix D.

The effect of supplying ten spares for each module (Case G) is examined in Table 3.12. Once more, the effect is more important in the LAIN availability, especially for $t > 60$.

Table 3.12 COMPARATIVE PERFORMANCES (CASE G; 10 SPARES)

	FCFS-S		FCFS-A		LAIN-S		LAIN-A	
t	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV	E[AV]	SDEV
10	49.50	1.27	50.00	2.43	49.62	0.78	50.00	1.16
20	46.44	3.36	47.00	4.16	49.54	0.86	50.00	1.40
30	43.64	3.84	44.10	4.27	49.01	1.22	50.00	1.38
40	41.04	3.98	41.83	4.71	48.32	1.60	49.36	1.83
50	39.49	4.11	40.17	4.89	47.39	1.93	47.76	2.39
60	38.57	4.10	39.07	4.49	46.44	2.09	46.37	2.25
70	37.94	4.22	38.52	4.47	45.38	2.15	45.16	2.73
80	37.51	4.05	38.03	4.23	44.41	2.21	44.10	2.39
90	37.40	4.06	37.73	5.05	43.43	2.29	43.18	2.52
100	37.10	4.02	37.44	4.81	42.67	2.26	42.66	2.77

IV. FURTHER DEVELOPMENTS

A. NON-CANNIBALIZATION

In all models considered so far it is assumed that working components belonging to inoperative aircraft (resulting from previous failures) can be used to substitute for modules which have just failed in another (otherwise operative) aircraft. This circumstance is what is known as "cannibalization", and it is usually implemented in real situations when it is necessary to maintain a high level of combat readiness through increased availability of vital assets for a limited period of time. As an example, the overall objective of the Chief of Naval Operations is to obtain at least seventy-two percent of fully-mission-capable aircraft in a squadron [Ref. 1:p.4].

In this section a modification of the original "analytic" model is developed for a situation where cannibalization is not employed. It is assumed that the time horizon of interest is relatively small, such that the adoption of a cannibalization policy does not affect significantly components' failure rates[1].

It is further assumed that an aircraft will return to its base almost as soon as the first vital component fails, and that no other component in this aircraft will suffer

[1]Experience shows that this kind of maintenance policy tends to decrease equipment life time if utilized for long periods. The effect is related to an increase of the individual failure rates, λ_i. This fact is corroborated by this author's own experience as head of the electronic equipments maintenance division in a Brazilian Navy FFG (Guided Missile Frigate).

failure until it is back in operation. This is, admittedly, a somewhat unrealistic assumption, but the lower bound it represents may be useful to demonstrate the dramatic role cannibalization plays in terms of increasing availability. The model may, also, serve as a quick tool for assessing the effect of this kind of maintenance policy. A model that supposes that degraded aircraft proceed with their mission can also be constructed, but this is not done at this time.

Let $A_{nc}(t)$ denote the total number of operational aircraft at each time t, for the non-cannibalization case. At $t=0$, let $A_{nc}(0)=A_c$, the number of initially deployed aircraft.

As is the case now, a failed component renders an aircraft inoperative until it is repaired, assuming there is no spare part in stock. Otherwise, the failed component is substituted, and will join the appropriate queue. Let $S_i = K_i - A_c$ be the number of spare parts available for module i.

$A_{nc}(t)$ may be represented by

$$A_{nc}(t) = A_c - \sum_{i=1}^{I} I_i(t) \times [N_i(t) - S_i] \tag{4.1}$$

where I, K_i, and $N_i(t)$ are as defined in chapter I, and $I_i(t)$ is the indicator variable defined by

$$I_i(t) = \begin{cases} 1, & if \;\; N_i(t) > S_i \\ 0, & if \;\; N_i(t) \leq S_i \end{cases} \quad . \tag{4.2}$$

1. A Numerical Example

Once again, a Monte Carlo simulation model is used to validate analytical results. The input data for the example used in this investigation are those of Table 3.2 in Chapter III.

In the simulation model, $A_{nc}(t)$ is updated only when a failure occurs or when a repair is completed, after considering inventory levels. When a failure occurs, the aircraft is brought back to the operational status if a spare exists, and the value of $A_{nc}(t)$ remains unchanged. Otherwise, the aircraft must wait until repair completion, and $A_{nc}(t)$ decreases by one. When a module is repaired, $A_{nc}(t)$ increases by one only if the component was causing an aircraft to remain inoperative.

With respect to the equations for $\beta_i(t)$ in (2.4), the only necessary modification is the use of $A_{nc}(t)$ (4.1) scaled by a in the place of $a_V(t)$. For the non-cannibalization case, only the **LAIN** policy is examined, and the corresponding results are compared to the usual (perfect cannibalization) plan, for both **FCFS** and **LAIN**.

The numerical example attempts to exploit a situation where the modules show large diversity in terms of failure and repair rates, and $p=30$ is used in (3.4).

Figure 4.1 displays a fairly satisfactory agreement between simulation and "analytical" results. Numerical values for the expected value of the aircraft availability as a function of t, $t\in[0,100]$ are shown for both cases.

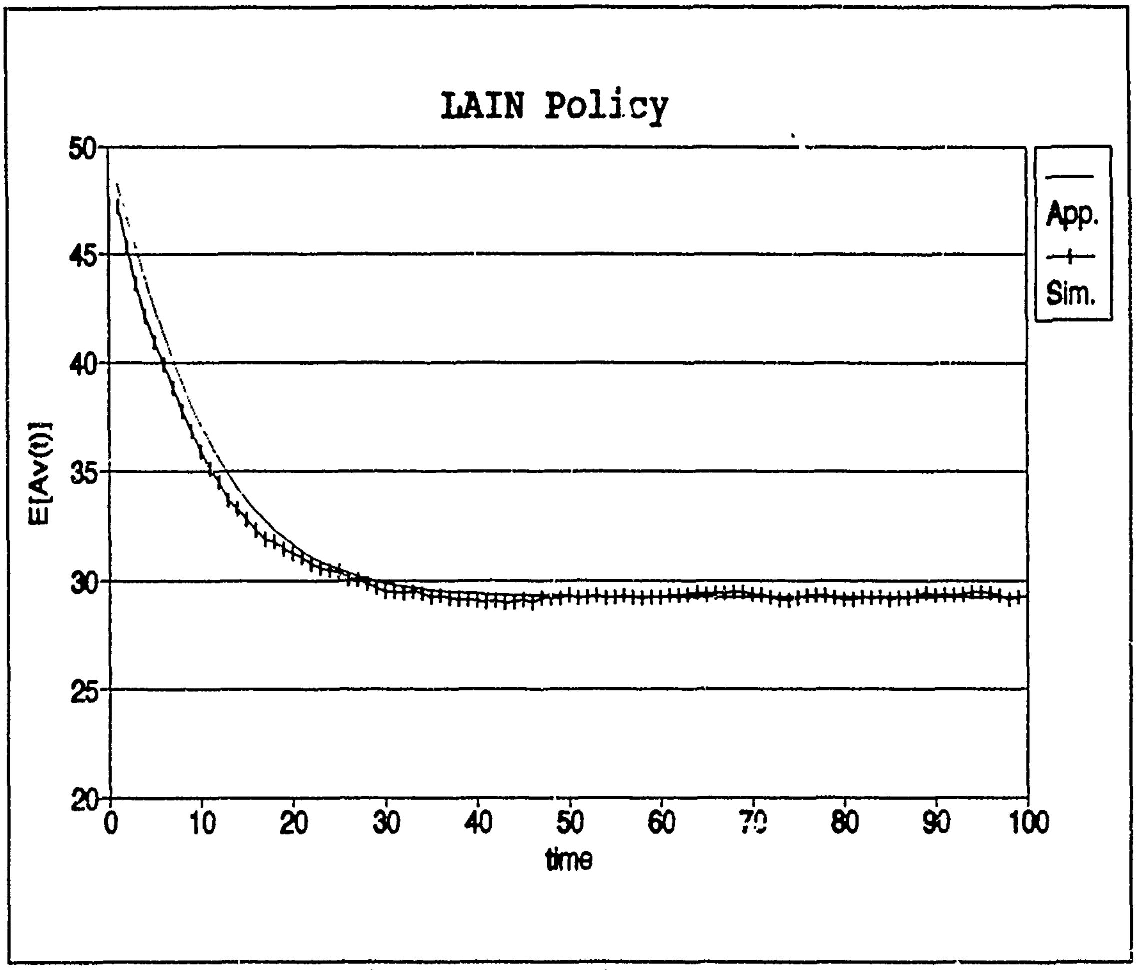

Figure 4.1 Non-Cannibalization LAIN Policy

Figure 4.2 exhibits the comparison among the various policies, showing the evident decrease in availability when cannibalization is not employed.

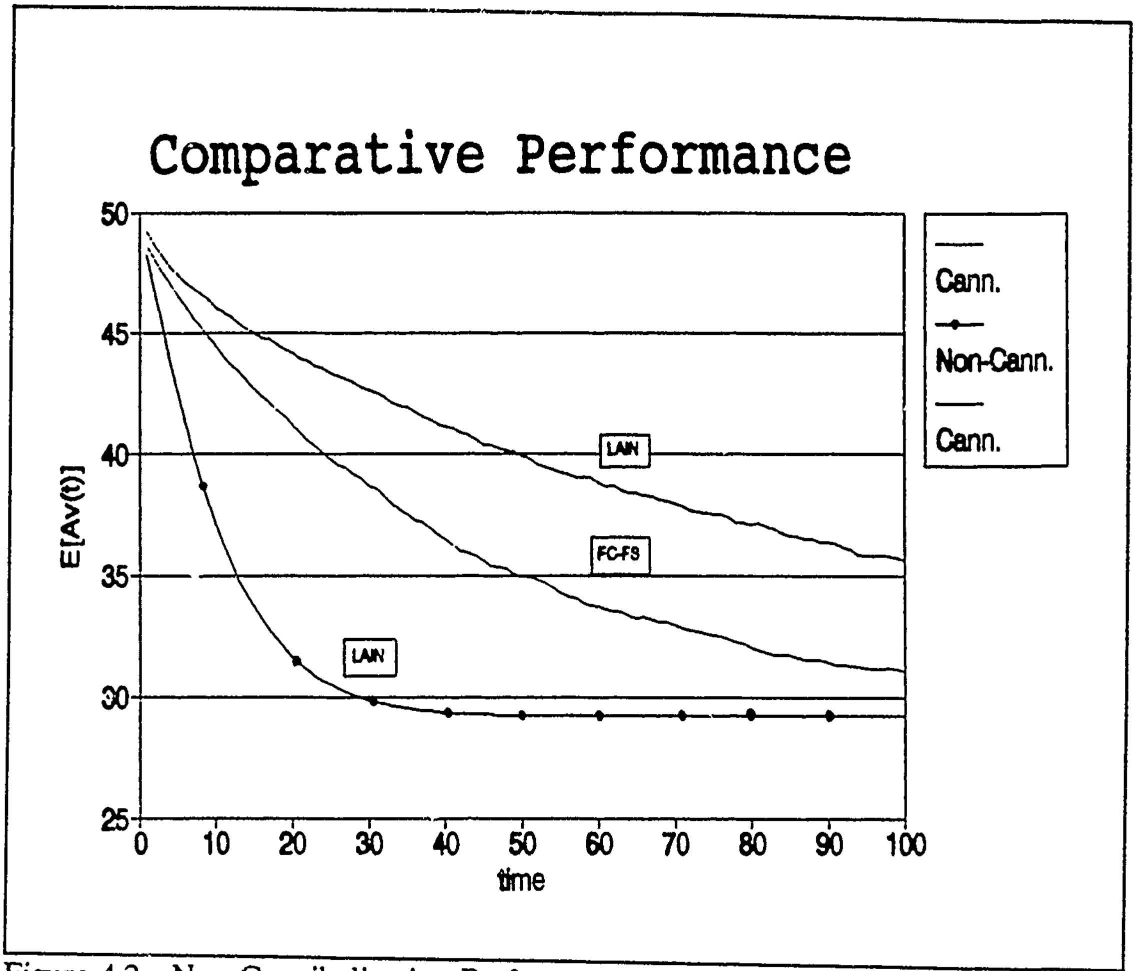

Figure 4.2 Non-Cannibalization Performance

Table 4.1 summarizes these results for $t=10,20,\ldots,100$. The numbers are rounded to the closest integer.

Table 4.1 NON-CANNIBALIZATION PERFORMANCE

TIME	LAIN (CANN)	FCFS (CANN)	LAIN(NO-CANN)
10	46	45	36
20	44	41	31
30	43	39	30
40	41	37	29
50	40	35	29
60	39	34	29
70	38	33	29
80	37	32	29
90	36	31	29
100	36	31	29

B. EXTERNAL REPAIR (BEYOND THE CAPABILITY OF (LOCAL) MAINTENANCE - BCM)

A more realistic characterization of the Air Unit -Repairman system must consider the case in which failed modules require a degree of support that is beyond the capability of local maintenance. The implication is that a certain portion of failed modules must be sent to a higher maintenance level, returning to the local level after an uncertain time. See Gaver, Isaacson and Pilnick [Ref. 9].

1. Pre-Local-Repair BCM

Suppose that every time a module of type j fails there is a probability η_j that the local repair shop will not be able to service it. This implies that this module has to be sent to another repair echelon, external to the deployed group. Denote these modules as BCM modules. Further assume that all BCM modules, after a random period of time, will return to the system.

The amount of time a BCM module of type j stays outside the system (to be repaired / substituted) is assumed to be an exponential random variable with mean $1/\delta_j$, i.e., BCM modules of type j return to the local level at a rate equal to δ_j. Let $K_j(t)$ be the time-dependent number of modules of type j available at the local level at time t, with $K_j(0)=K_j$, K_j-$K_j(t)$ BCM modules at time t, $j=1,2,...,I$.

For all $j=1,..,I$, the probability $P_j(\mathbf{N}(t)) = P_j(t)$ that a module of type j fails in the interval $(t,t+dt)$ and is repairable at the local repair facility (non-BCM), conditional on the present state $\mathbf{N}(t)$ is given by

$$P_j(t)\,dt = (1 - \eta_j)\,\lambda_j\,[K_j(t) - N_j(t)]\,dt + o(dt) \tag{4.3}$$

Now define $\alpha_j(t) \approx K_j(t)/a$, and substitute $\alpha_j(t)$ for α_j in (2.5) for the definition of $a'_V(t)$.

The following systems of differential equations may be written for the approximate scaled mean of $\mathbf{N}(t)$, $\beta(t)$, and for the scaled number of modules at the local level $\alpha(t)$:

$$\frac{d\beta_j(t)}{dt} = \lambda_j(1-\eta_j)\,a'_V(t) - \mu_j p_j(t) \tag{4.4}$$

$$\frac{d\alpha_j(t)}{dt} = \delta_j(\alpha_j - \alpha_j(t)) - \lambda_j \eta_j a'_V(t) \tag{4.5}$$

a. *A Numerical Example*

Consider the sample system depicted in Chapter III-C (Table 3.3), and suppose that all BCM modules return to the local level at a rate $\delta_i = 1/30$, $i = 1,2,...,10$. Further assume that the proportion of BCM modules of type i, η_i, is equal to 0.20 for all i. Figure 4.3 displays the mean aircraft availability for $t \in [0,100]$, calculated according to (4.4) and (4.5) for both the FCFS and LAIN policies. The plot includes simulation results, and it is clear that the accuracy of the analytical model is very acceptable for this case.

The correctness of the model was tested for different values of η_i. However, numerical and analytical difficulties are observed for some particular parameter values. A possible explanation may be that the decrease in local demand weakens the heavy traffic condition necessary for the present formulation. In Appendix B this subject is further explored.

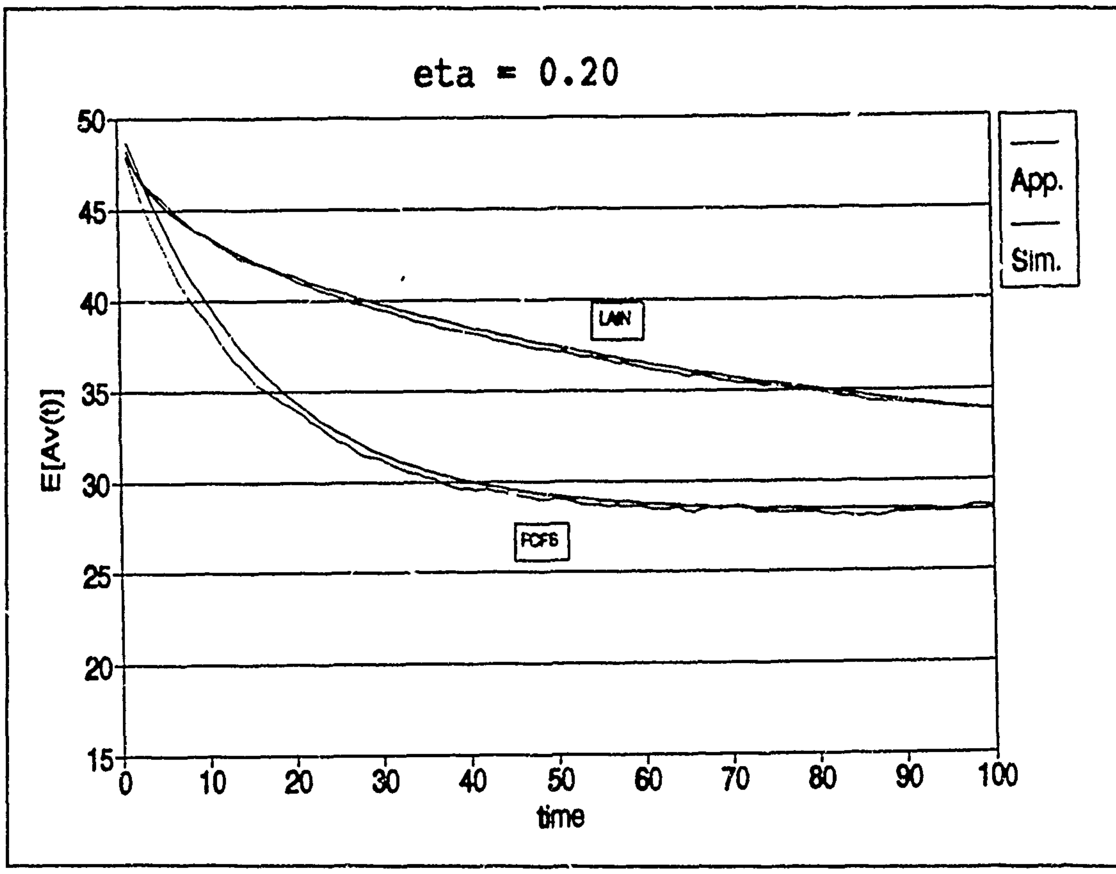

Figure 4.3 Pre-Local-Repair BCM

2. Post-Local-Repair BCM

Gaver, Isaacson and Pilnick [Ref. 9] conceived an interesting modification to the pre-local-repair BCM case. They argue that immediate consignment to external repair of a failed module is, to say the least, an optimistic assumption. If every failed module must be first submitted to the local test and repair cycle, and only at termination it is either completely repaired or a decision for external repair is made, they model the modified system using

$$\frac{d\beta_j(t)}{dt} = \lambda_j a'_V(t) - \mu_j p_j(t) \tag{4.6}$$

$$\frac{d\alpha_j(t)}{dt} = \delta_j(\alpha_j - \alpha_j(t)) - \mu_j \eta_j p_j(t) \tag{4.7}$$

a. *A Numerical Example*

Once more consider the system described in Table 3.3, and suppose that $\delta_i^{-1} = 30$ for all *i*. Figure 4.4 and 4.5 show a useful accurate agreement between simulation and analytical results for different values of η_i.

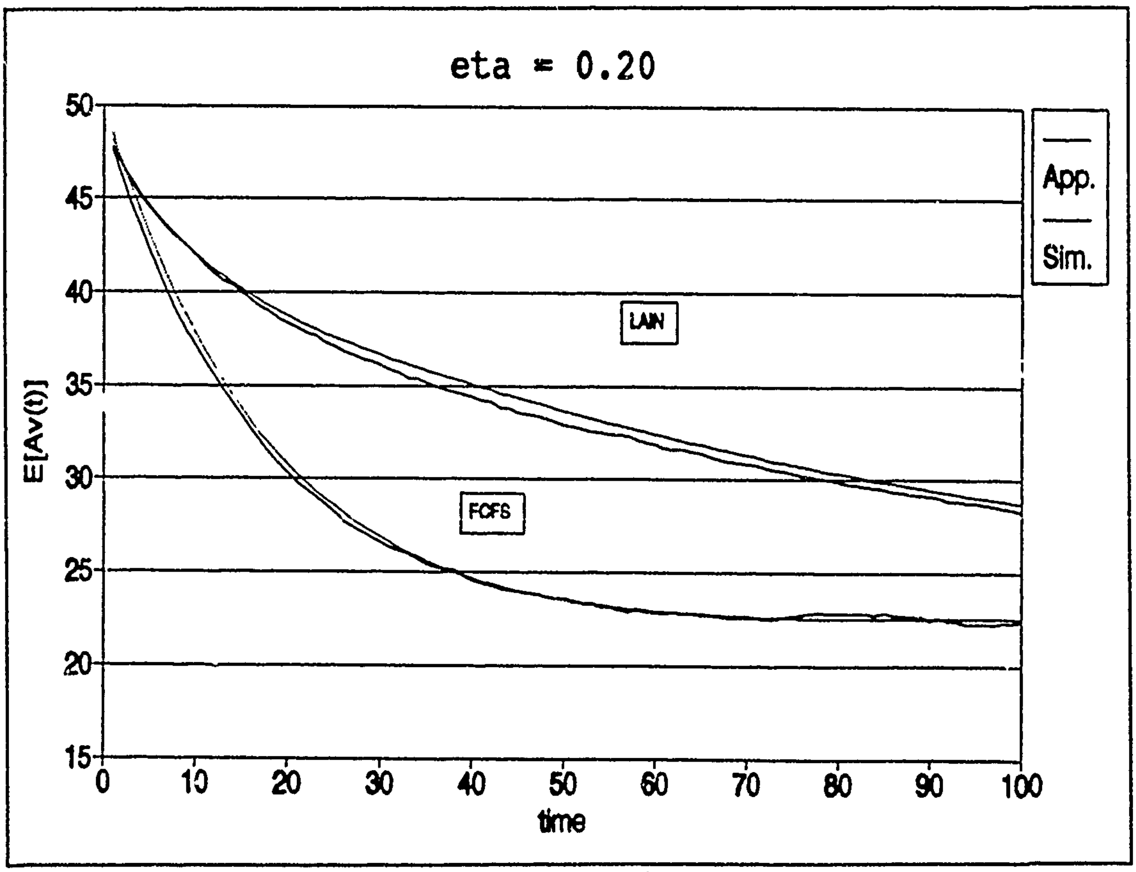

Figure 4.4 Post-Local-Repair BCM (η = 0.20)

For the post-local-repair case, numerical and/or analytical complications were not observed when the model was tested under a considerable number of parameter combinations.

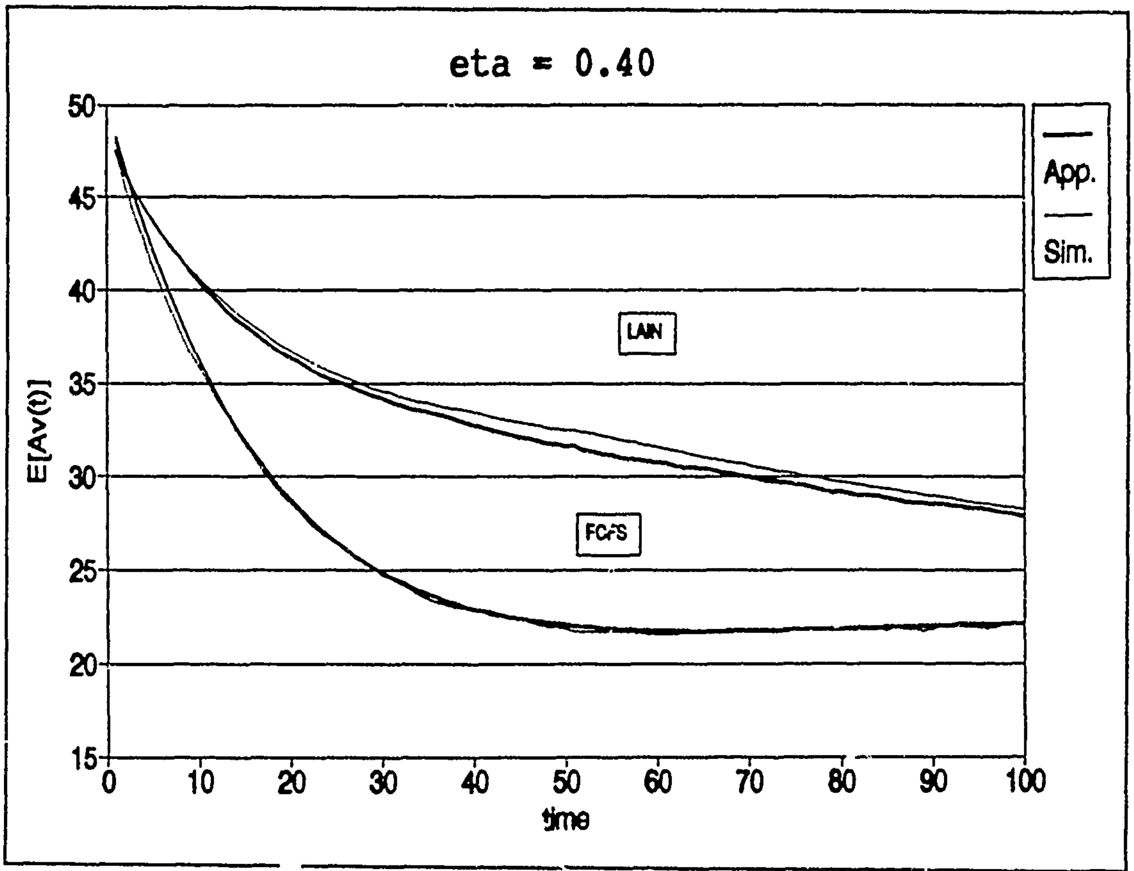

Figure 4.5 Post-Local-Repair BCM (η = 0.40)

V. OPTIMIZATION MODELS

In this chapter two simple optimization models are described for the perfect cannibalization, non-BCM case. The models attempt to represent different decision problems, and can be readily adapted for different situations, such as BCM. One of the basic assumptions here is that an adaptive scheduling policy (LAIN) is used.

Every stochastic optimization problem must reflect an attitude toward risk; it is therefore often possible to pass from one type of formulation to an equivalent one. The models described here are designed for planning purposes, in the sense that the decision does not depend in any way on future observations of the random vector $N(t)$. Such models are usually called *anticipative models*; see, for example, R. J.-B. Wets [Ref. 14].

A. DEFINITIONS

In order to describe the optimization models for the combat availability problem, a larger set of variables must be defined. The complete set of variables is defined below:

Let

I = number of distinct mission-essential component types in each a/c;

K_i = initial provision of components of type i, including those installed in the aircraft ;

c_i = cost of one unit of component type i ;

A_c = number of aircraft initially deployed ;

S_i = spare parts provided for component of type i, i.e., $S_i = K_i - A_c$;

T = time horizon ;

P_{min} = minimum percentage of the original number of a/c required at time T;

$s(t)$ = $argmin\ \{K_i(t) - N_i(t)\}$;

$A_V(t)$ = $min\ \{A_c, K_s - N_s(t)\}$;

A_{min} = $P_{min} \times A_c$;

$N_i(t)$ = number of components of type i being repaired or waiting in queue at time t ;

$\beta_i(t)$ = approximation of the scaled mean of $N_i(t)$;

$\Sigma(t)$ = scaled variance-covariance matrix of $\mathbf{N}(t)$, and

a = $A_c + \Sigma_i K_i$.

As was shown in chapter III, $N_i(t)$ is approximately normally distributed, i.e.,

$$N(t) \approx \mathbb{N}(a.\beta(t), a.\Sigma(t)) \quad . \tag{5.1}$$

where $\mathbf{N}(t)$ is the vector with components $N_i(t)$, and $\beta(t)$ has components $\beta_i(t)$, $i = 1,2,...,I$.

B. RELIABILITY MODEL

Suppose that, during the planning phase of a combat mission that is expected to last T units of time, it has been established that a minimum percentage P_{min} of the initial number of fully mission capable aircraft is required with high probability, r. Then, assuming that resupply is not possible during the mission, it seems reasonable to consider only the combat availability at time T. Further imagine that a general stockage policy for many missions is under study, so that costs must be minimized for this particular mission.

A general mathematical description of this problem is

$$\begin{aligned} \min \quad & \sum_i c_i S_i && (i) \\ st. \quad & P\left[A_V(T) \geq A_{V_{\min}}\right] \geq r \;\; , \;\; 0 < r < 1 && (ii) \\ & S_i \in \{0,1,2,\ldots\} \quad . \end{aligned} \tag{5.2}$$

We can interpret (5.2) as a wish to minimize spares allocation cost subject to a minimum system reliability, where the system is now viewed as the whole Air Unit.

In order to correctly employ the approximation in (5.1), it is necessary to examine the probabilistic (or chance) constraint (*ii*) in (5.2) with respect to the corresponding expression for each item. Consider

$$\begin{aligned} A_V(T) &= \min_i \{A_c, K_i - N_i(T)\} \\ &= \min_i \{A_c, (A_c + S_i) - N_i(T)\} \\ &= \min \{A_C, (A_C + S_{s(T)}) - N_{s(T)}(T)\} \quad . \end{aligned}$$

Now, under heavy traffic and for large enough T, so that combat availability is determined by the module with the least availability, i.e.,

$$A_V(T) = A_C + S_{s(T)} - N_{s(T)}(T) \quad ,$$

the event $A_V(T) \geq Av_{min}$ can be rewritten as

$$(A_c + S_{s(T)}) - N_{s(T)}(T) \;\geq\; A_{V_{min}}$$

$$\therefore \; N_{s(T)}(T) \;\leq\; (A_c + S_{s(T)}) - A_{V_{min}} \quad .$$

and, using (5.1), we have approximately

$$P\,[N_{s(T)}(T) \leq (A_c + S_{s(T)}) - A_{V_{min}}] \;\approx\; \Phi \left[\frac{(A_c + S_{s(T)}) - A_{V_{min}} - \; a.\beta_{s(T)}(T)}{a \,.\, \sigma_{s(T)}(T)}\right]$$

where $\sigma_{s(T)}(T)$ is the diagonal element of the matrix $\Sigma(T)$ corresponding to the module $s(T)$.

A deterministic approximation for the model in (5.2) is, then

$$\begin{aligned} \min \quad & \sum_i c_i S_i \\ st. \quad & S_{s(T)} \geq z_r\, a\, \sigma_{s(T)}(T) + A_{V_{min}} + a\, \beta_{s(T)}(T) - A_c \\ & S_i \geq 0 \qquad , \end{aligned} \tag{5.3}$$

where z_r is the r^{th} quantile of the standard normal distribution.

Note that (5.3) is a continuous approximation of the actual discrete problem (5.2), and that, even though the constraint in (5.3) is described with a single inequality, in practice its computation at each stage of the optimization procedure involves the solution of the system of differential equations used to define $\beta(t)$ and $\Sigma(t)$.

C. EXPECTED VALUE MODEL

It is common practice in stochastic programming models to make use of expected values when the coefficients of the decision variables are random. In the following simple model we seek to achieve the maximum expected value of combat availability at T, subject to a budget constraint:

$$\begin{aligned} \max \quad & E[A_V(T)] \\ st \quad & \sum_{i=1}^{I} c_i S_i \leq B \\ & S_i \in \{0,1,2,\ldots\} \ , \end{aligned} \tag{5.4}$$

where B is the budget.

The approximation to the objective function in (5.4)

$$E[A_V(T)] \approx \min\{A_c, (A_c + S_1) - a\beta_1(T), \ldots, (A_c + S_I) - a\beta_I(T)\}$$

may,once again, be simplified to

$$(A_c + S_{s(T)}) - a\beta_{s(T)}(T)$$

under the same conditions described in the last section.

The continuous approximation to the problem (5.4) is, then

$$\begin{aligned} \max \quad & (A_c + S_{s(T)}) - a\beta_{s(T)}(T) \\ st \quad & \sum_{i=1}^{I} c_i S_i \le B \\ & S_i \ge 0 \ . \end{aligned} \tag{5.5}$$

VI. CONCLUSIONS AND RECOMMENDATIONS

In this thesis several mathematical models for evaluating scheduling and spares stockage rules in a transient dynamic combat environment are reviewed, developed and analyzed for a situation in which a large population of modules under heavy traffic conditions drives combat availability. Even though this paper specifically considered an Air Unit detachment problem, it is clear that these models may be utilized in numerous environments.

The adaptive priority policy (LAIN) was shown to considerably improve aircraft availability when compared to a standard FCFS scheme. The modeling technique using a diffusion approximation provides speedy solutions for a wide range of problems, and it provides the means for ready comparison of alternative scheduling policies, reflecting diverse organizational maintenance disciplines.

Initial results suggest that a pure priority scheme, based on the least available item at each time, overperforms arbitrary weighting procedures. This subject must be further explored.

The model for situations where cannibalization is not employed may help a decision maker, through a quick demonstration of the dramatic effect of this policy in terms of combat availability.

The situation in which failed modules require a degree of support that is beyond the capability of the local echelon is readily accounted for with small modifications to the original model. In this thesis we describe these modifications for the mean availability only. An area for further research is the development of equations for the variance of these BCM systems. Another point of interest is the development of models that take into account some mixture of pre- and post-local-repair allocation to distant repair, as well as possible mistakes in these consignments.

The optimization models described here are an attempt to apply the analytical models as a framework for choosing spare modules allocations in various environments. The actual solution of these models is certainly a challenging continuation of the present work.

APPENDIX A. GRAPHICS FOR THE CASE STUDY

In this appendix, a series of plots are used to demonstrate graphically the precision that is achieved by the diffusion approximation models. Cases A through D in Chapter III-C are reproduced here. For each case, the following plots are provided:

1. FCFS vs. LAIN - the expected combat availability for both policies is graphed for $t \in [0,100]$. Simulation results are also plotted, for comparison;

2. FCFS policy - lower and upper probability levels are shown, together with the mean availability. These levels are computed as the expected value of combat availability ($E[A_V(t)]$) $\pm$ the standard deviation (s) times the 95^{th} percentile of the standard normal distribution (1.65). Simulation values are also plotted;

3. LAIN policy - as in item 2 above.

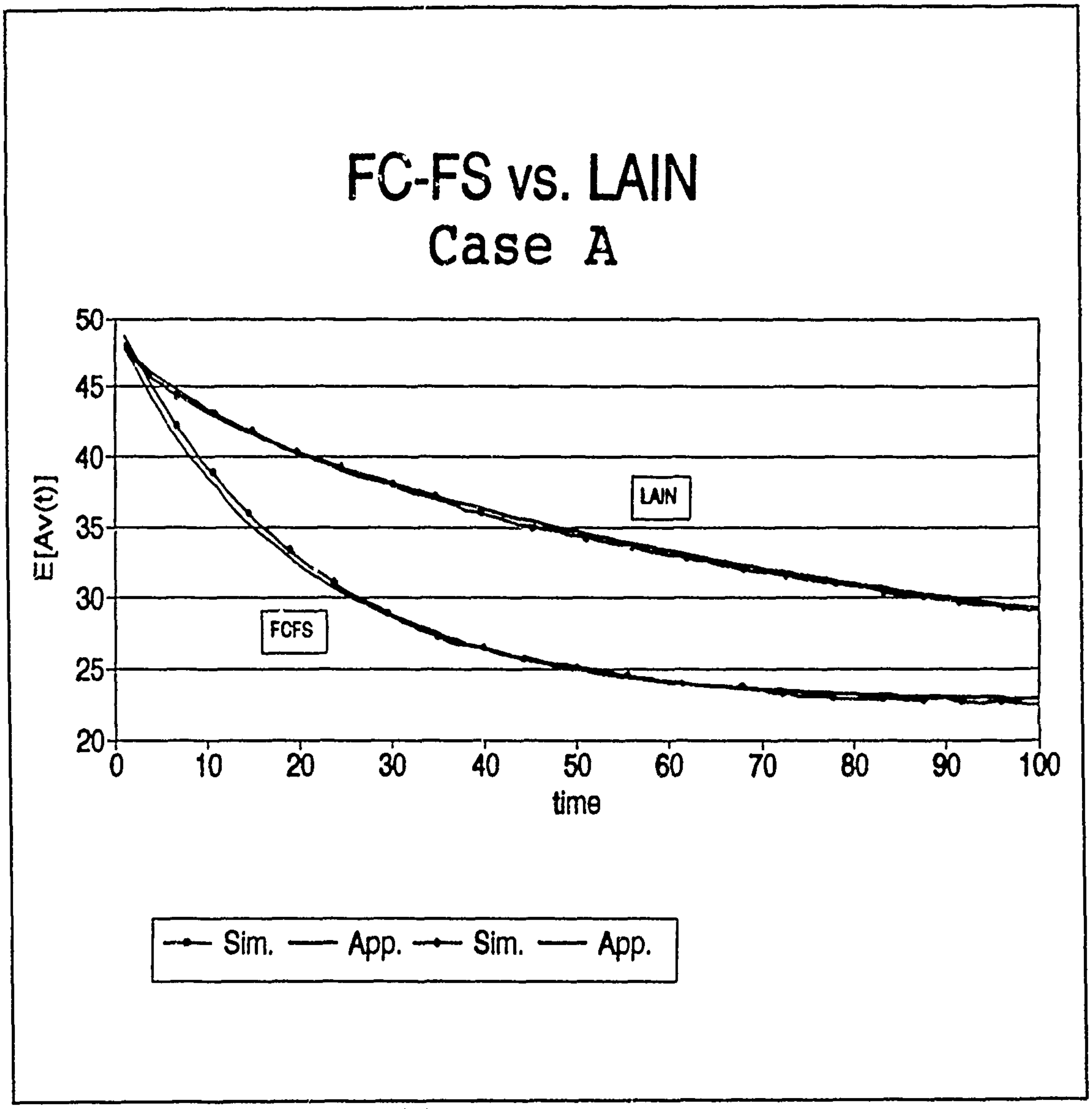

Figure A1. Case A; Both Policies

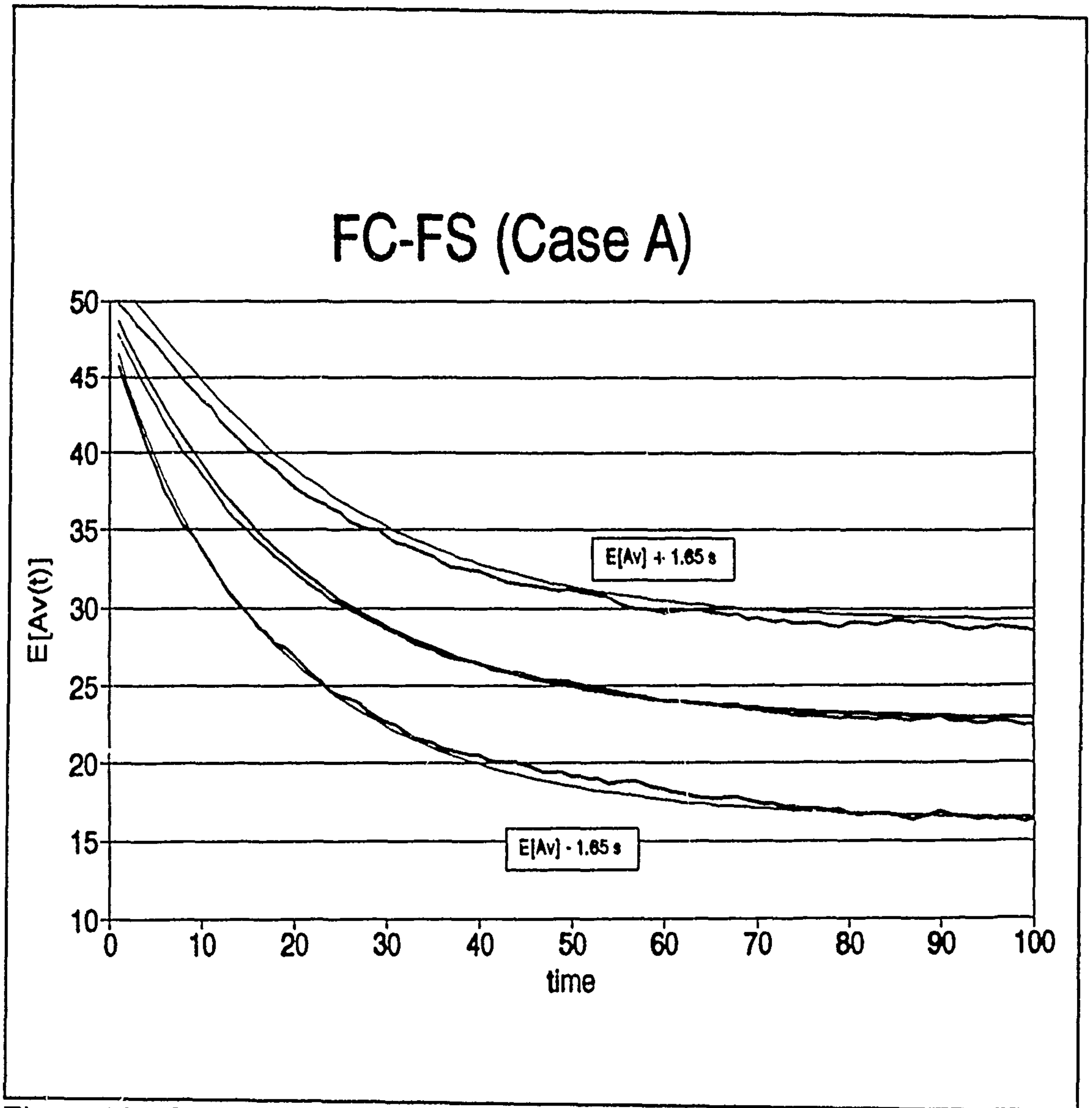

Figure A2. Case A; FCFS

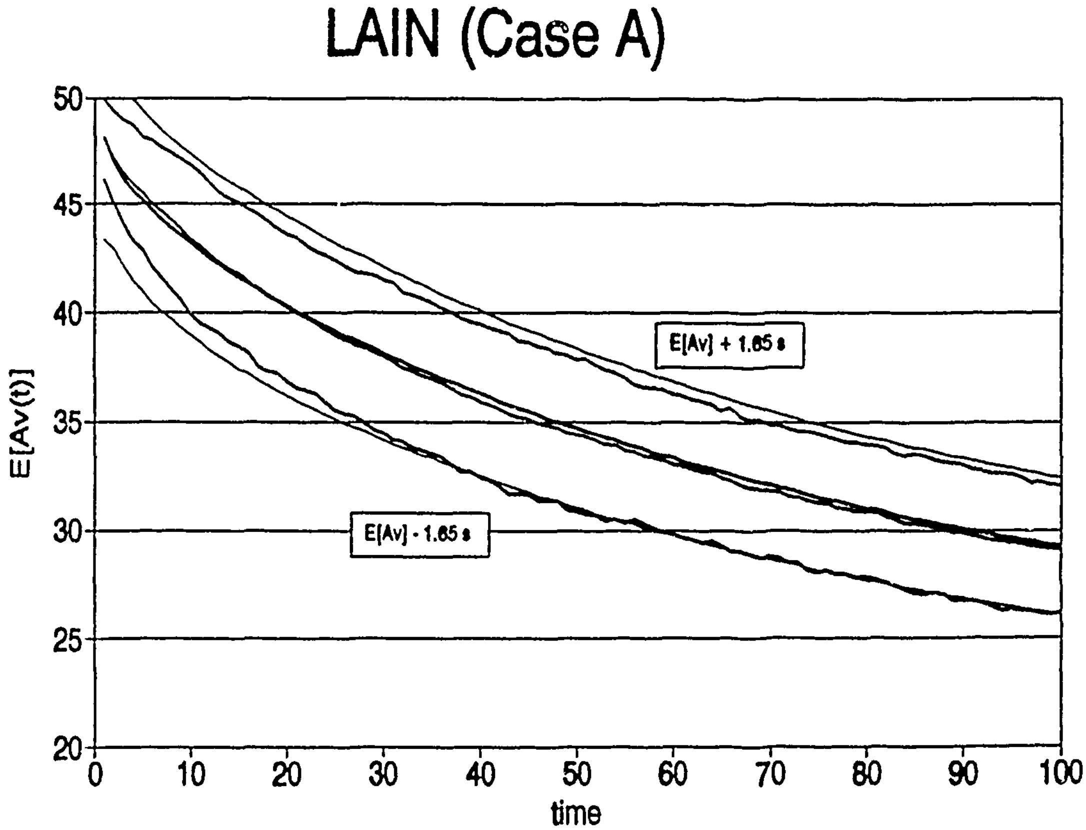

Figure A3. Case A; LAIN

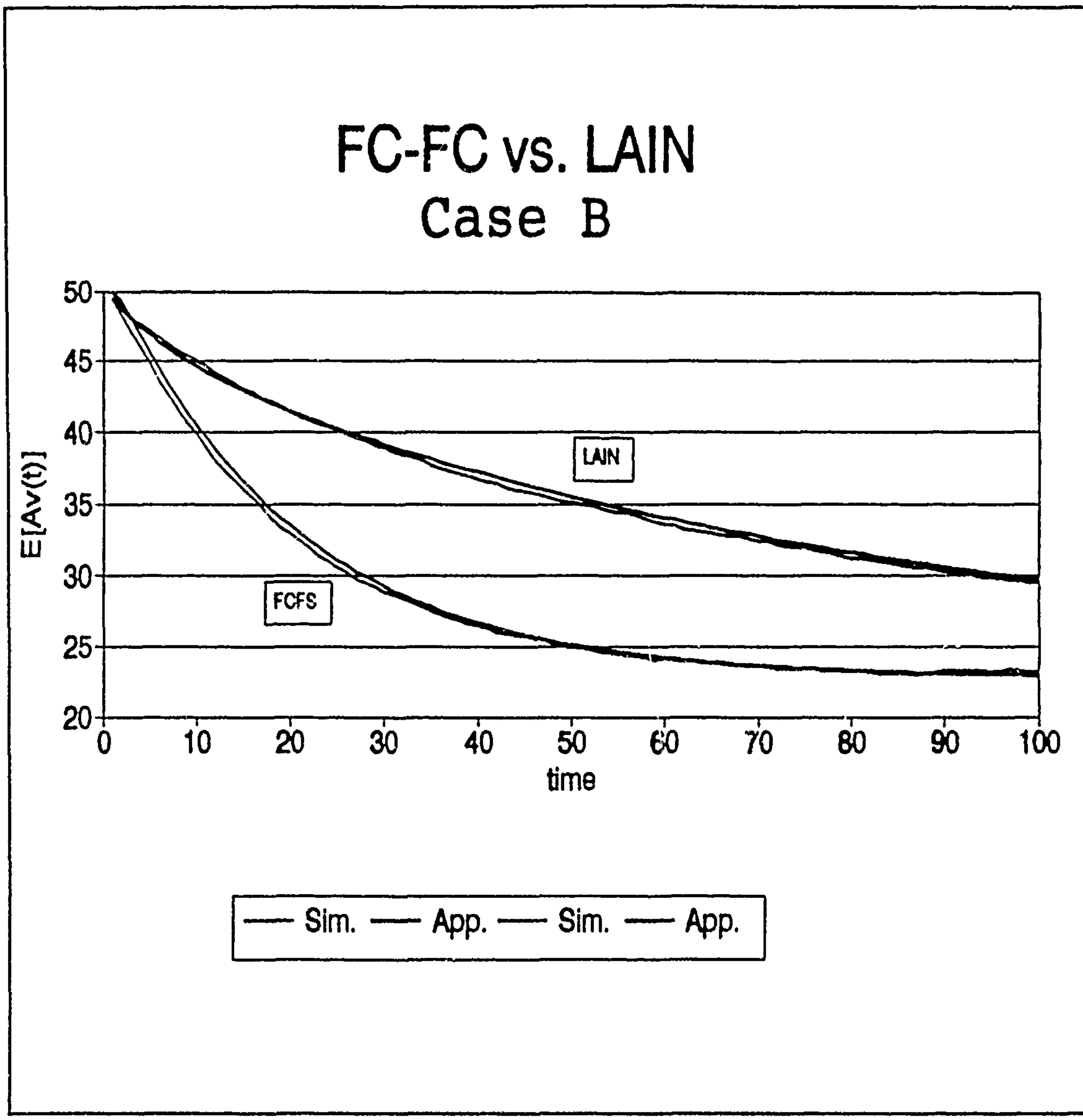

Figure A4. Case B; Both Policies

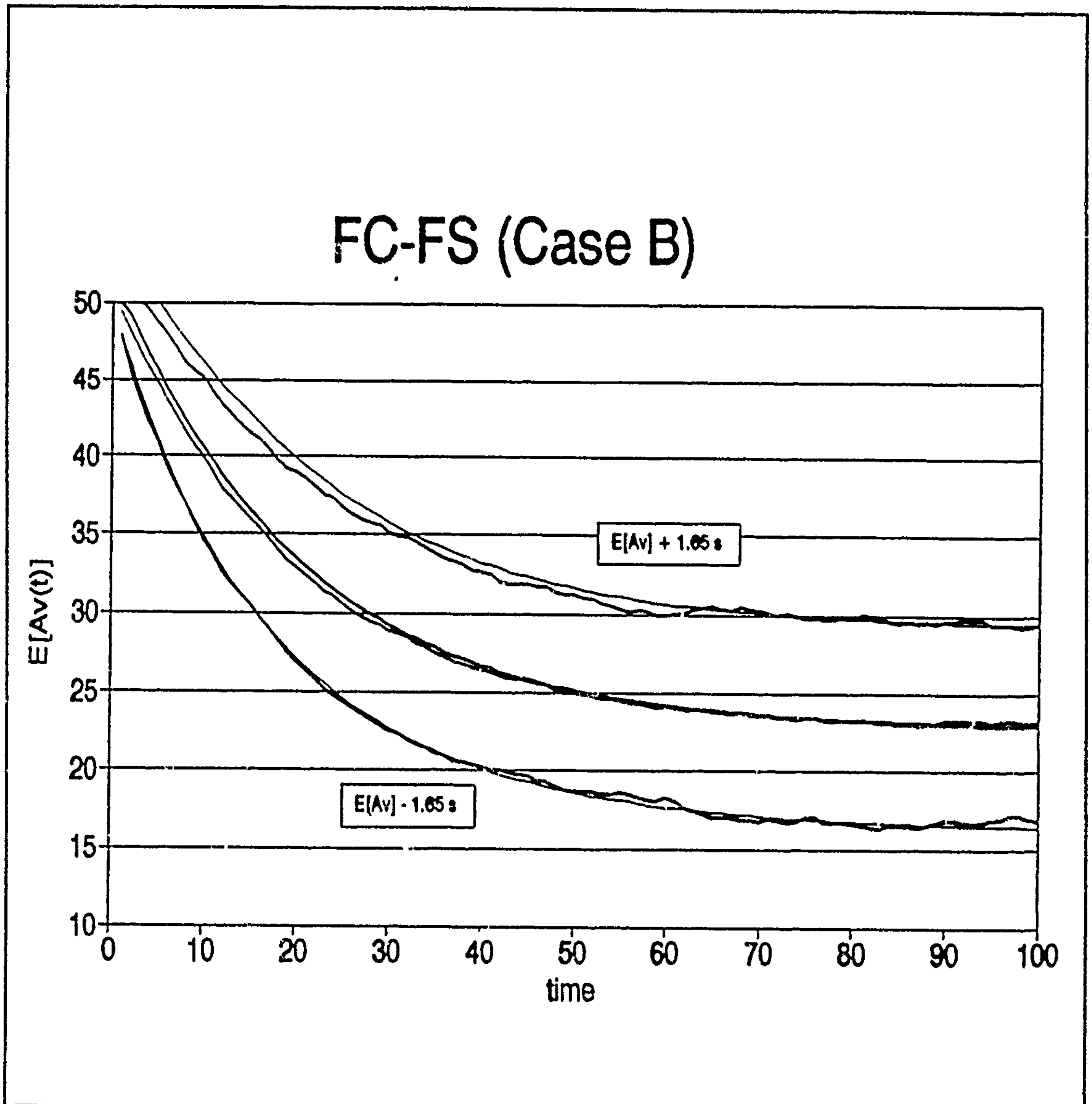

Figure A5. Case B; FCFS

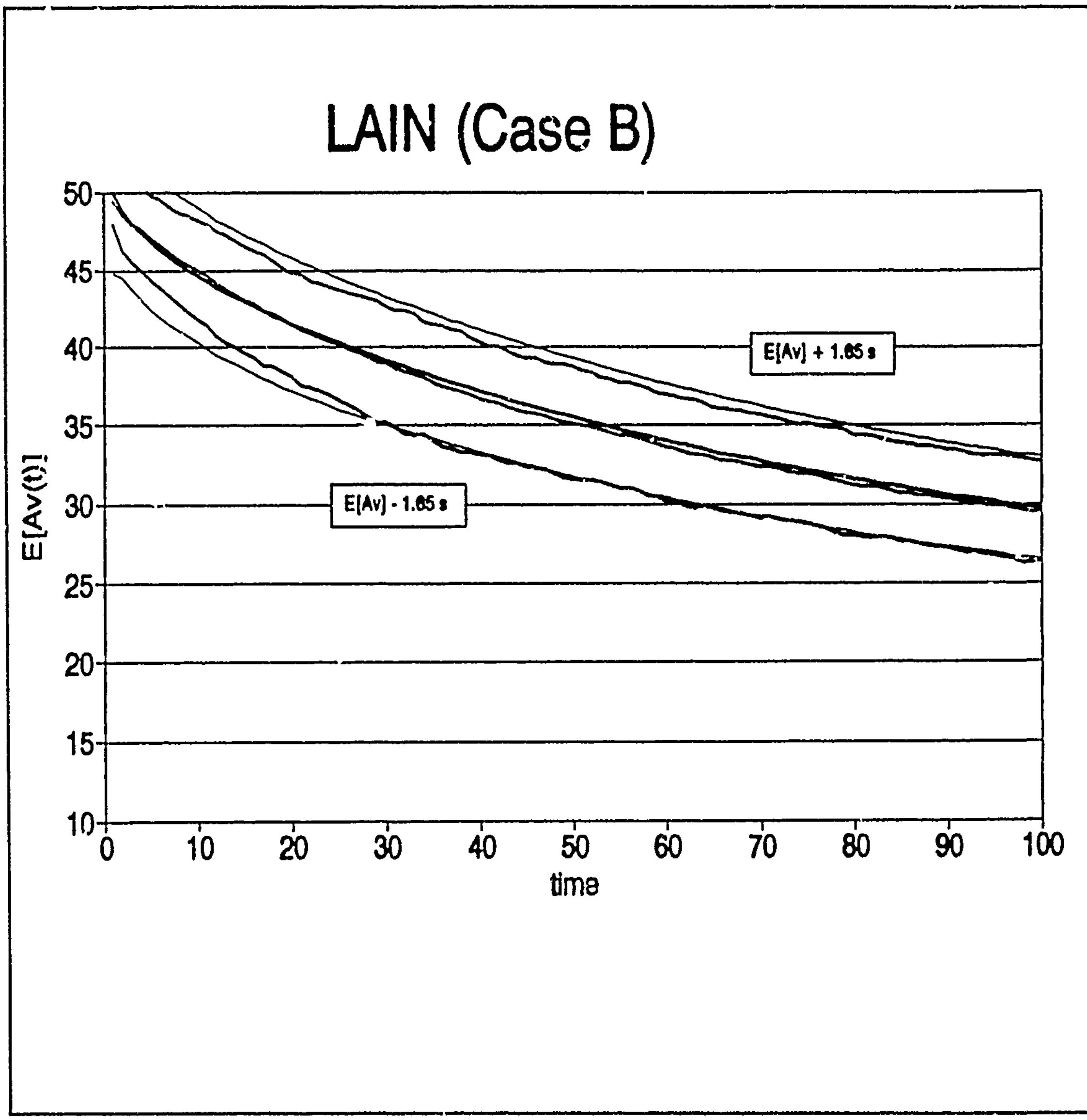

Figure A6. Case B; LAIN

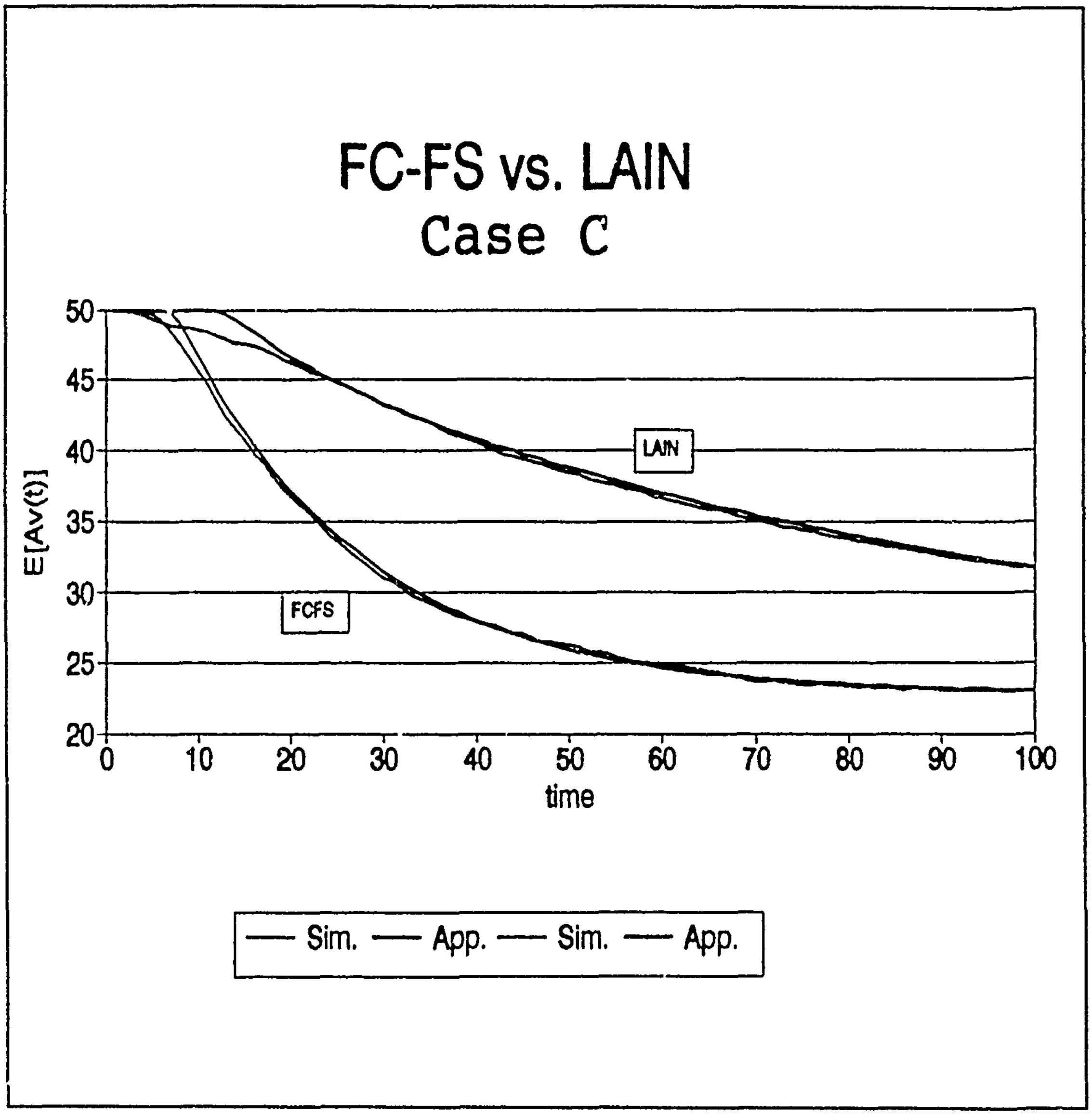

Figure A7. Case C; Both Policies

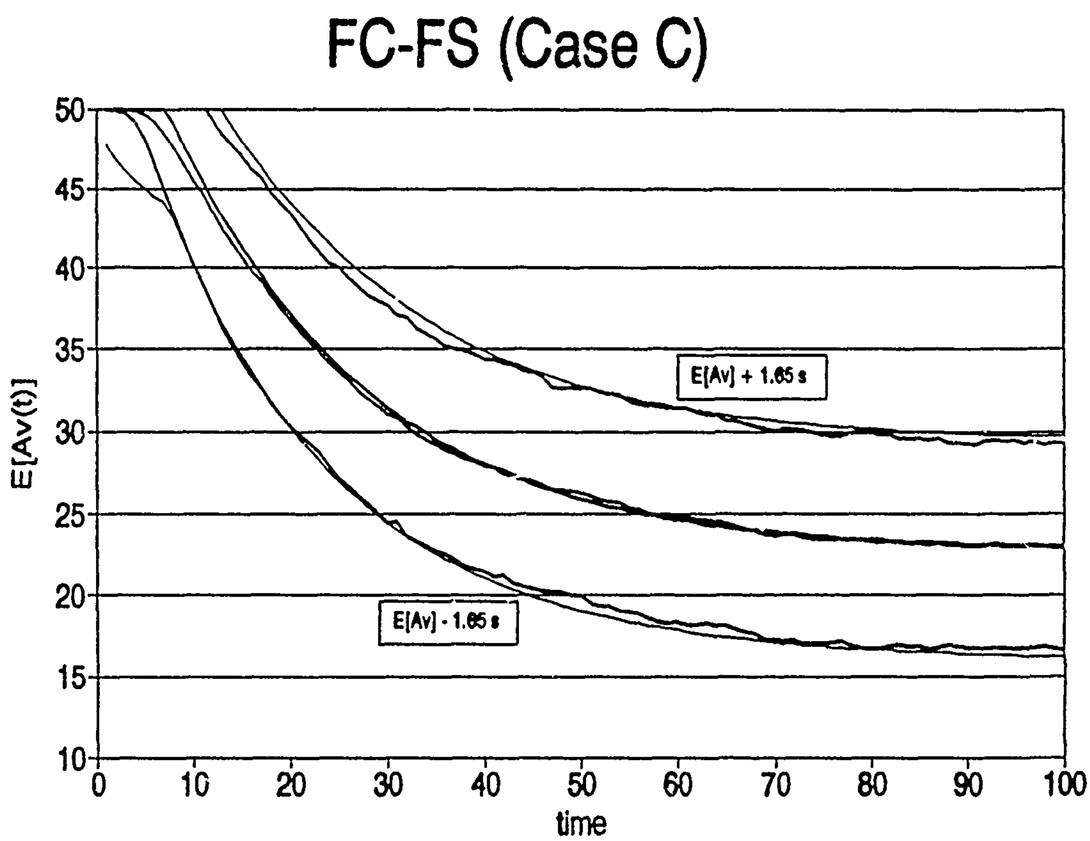

Figure A8. Case C; FCFS

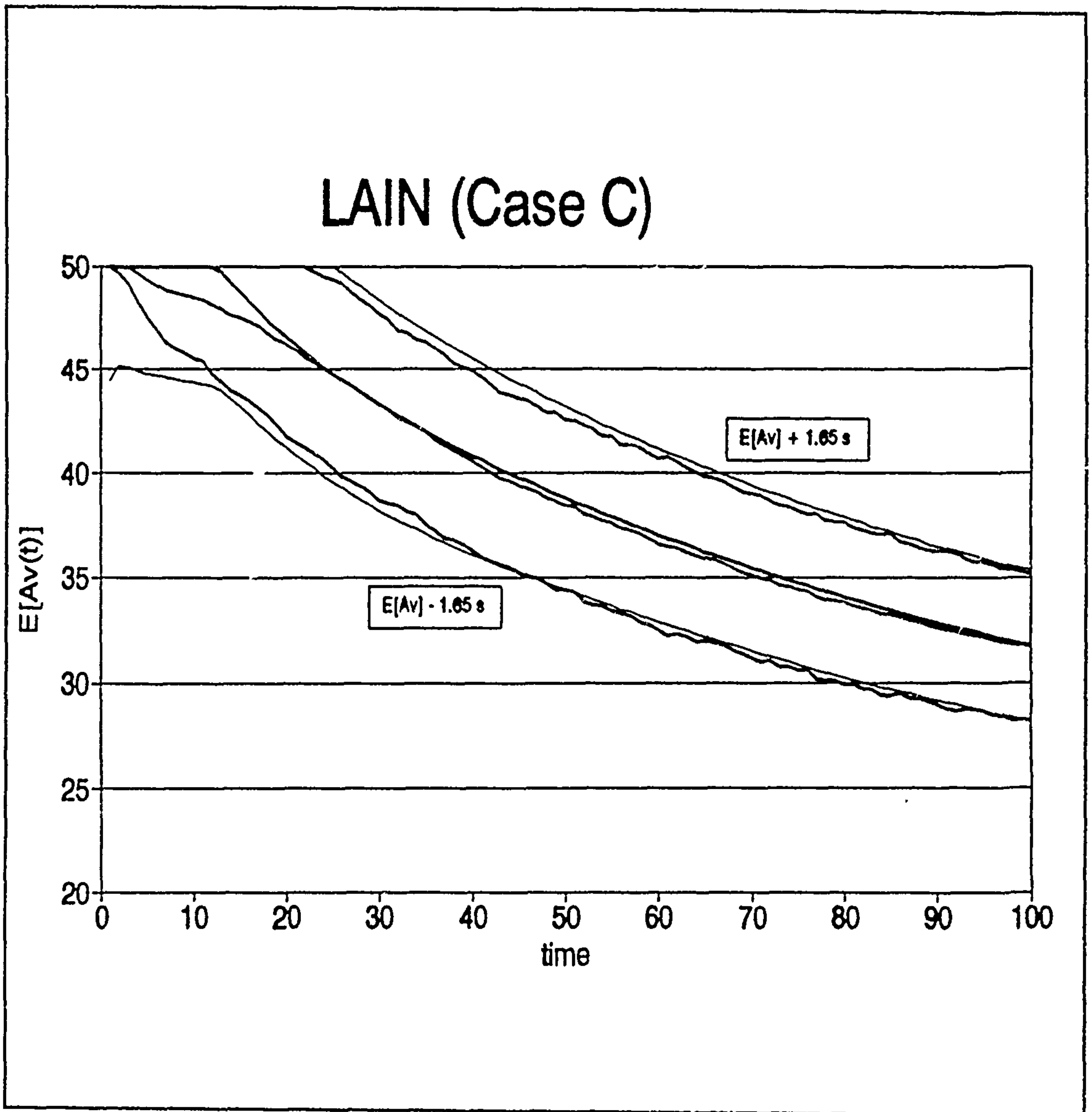

Figure A9. Case C; LAIN

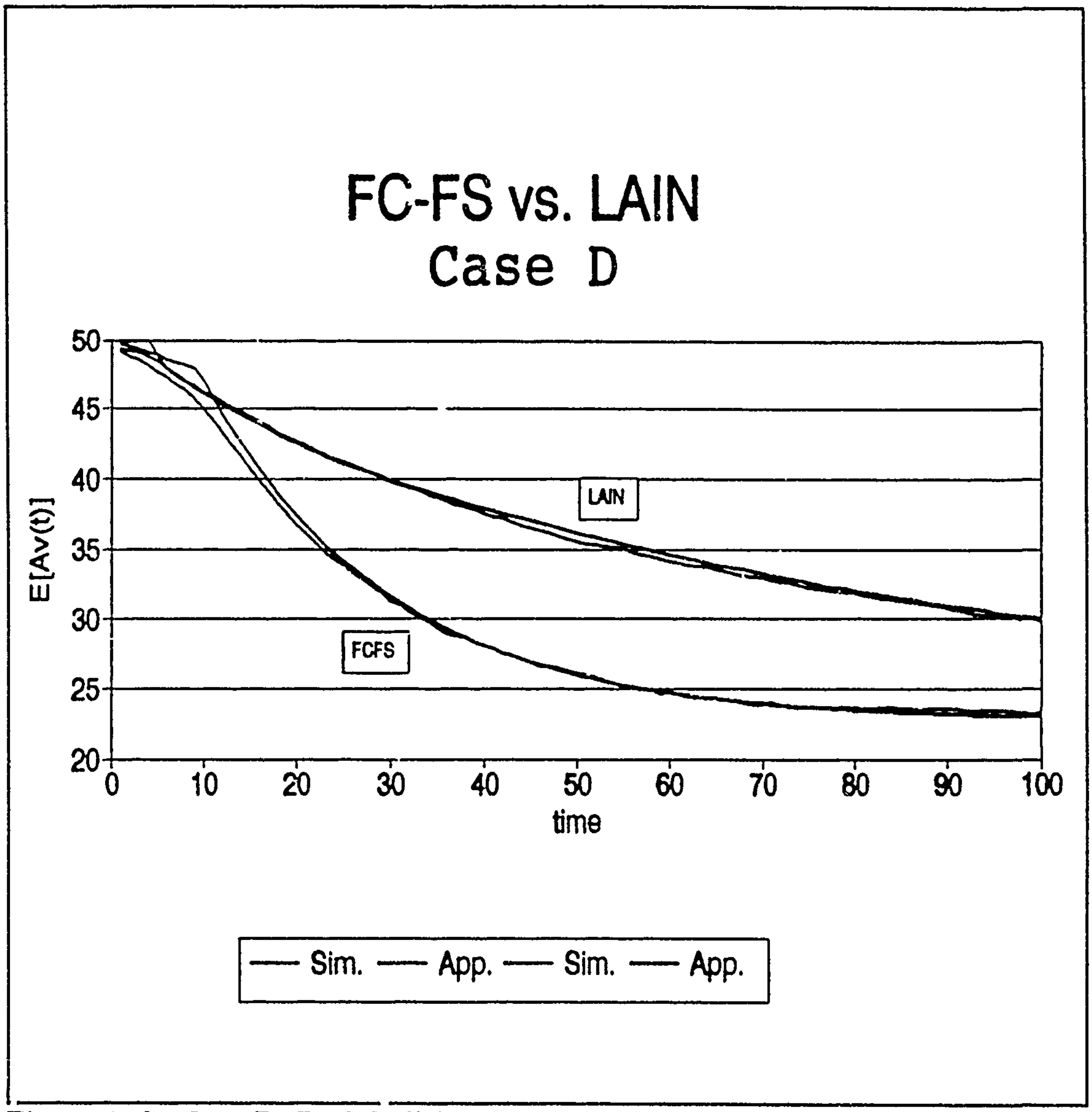

Figure A10. Case D; Both Policies

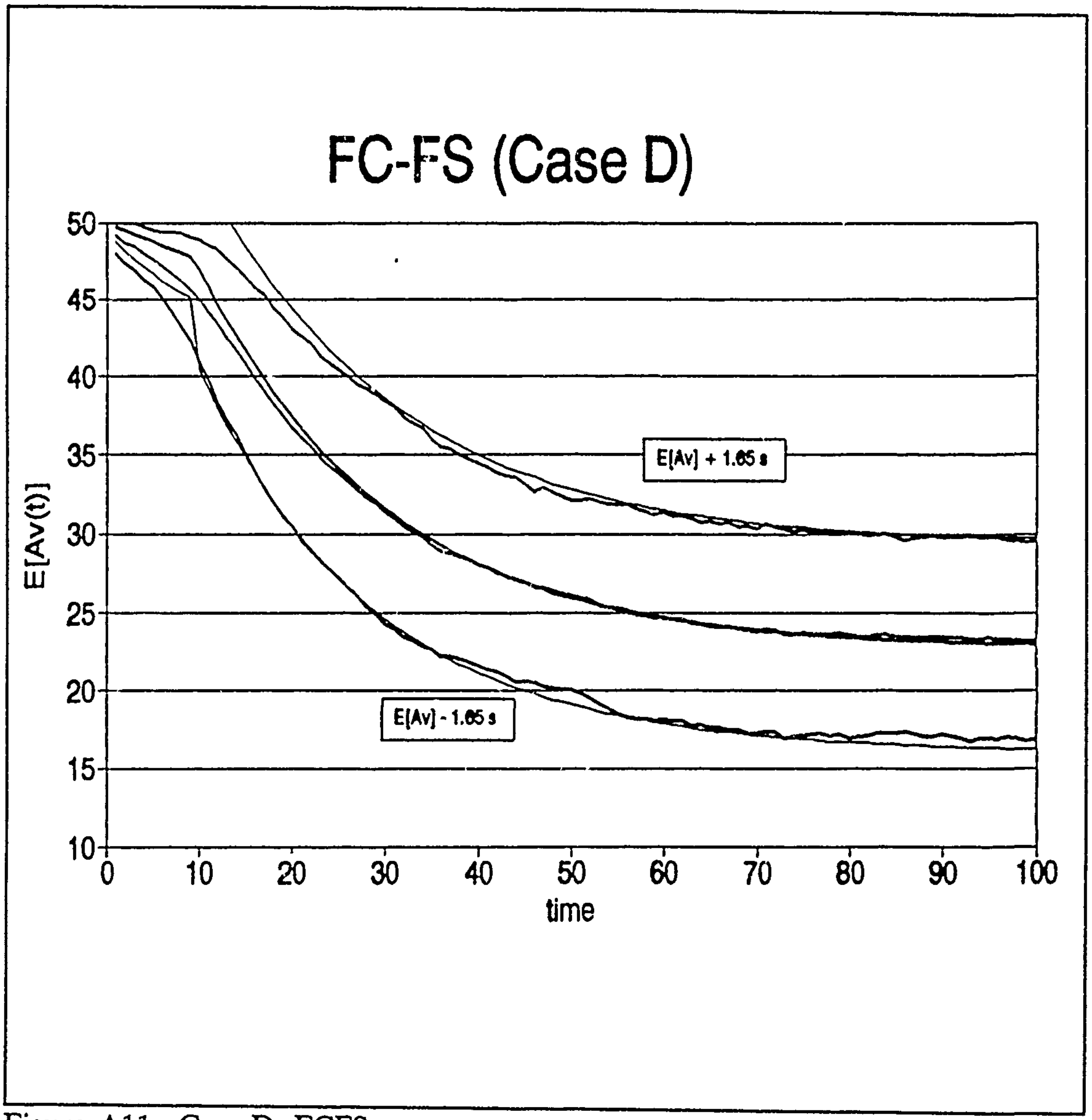

Figure A11. Case D; FCFS

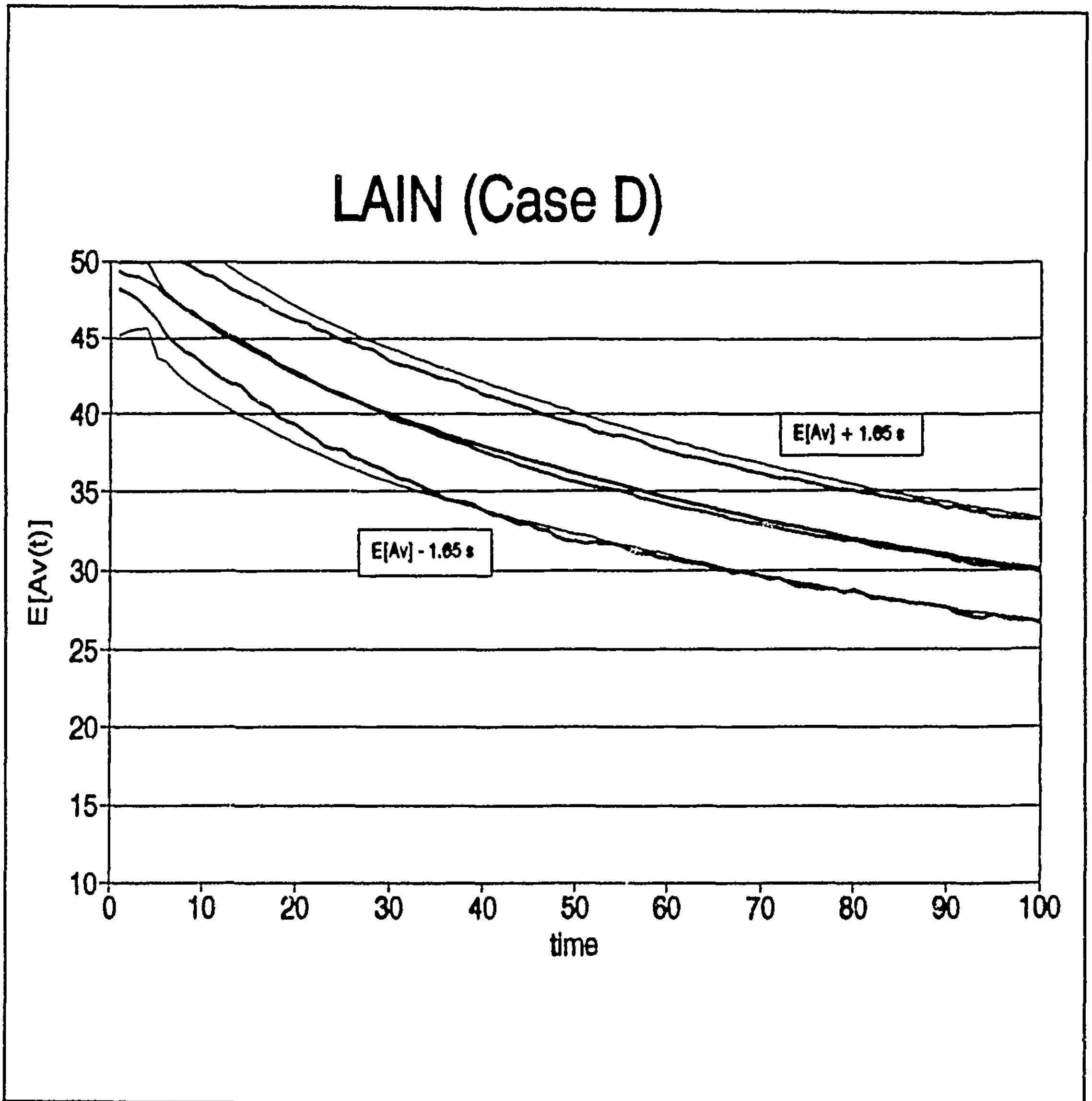

Figure A12. Case D; LAIN

APPENDIX B. PRE-LOCAL-REPAIR BCM

In this appendix, some of the numerical/analytical difficulties that occur in the pre-local-repair BCM model in (4.4) and (4.5) are summarized graphically. The model was extensively tested by means of a very accurate ODE solver developed by W. B. Gragg (Naval Postgraduate School) for the MATLAB-PC package [Ref. 13].

The model was tested for several combinations of values for the parameters η and δ. Figure 4.3 in Chapter IV displayed very accurate results when $\eta_i = 0.20$ and $\delta_i^{-1} = 30$, $i = 1,2,...,10$. For some parameter values, however, even stiff ODE solvers fail to achieve convergence at certain time intervals.

Figure B1 shows the result of applying our model to the sample system characterized by Table 3.3 in Chapter III, with $\eta_i = 0.30$ and 0.40, respectively. It is clear that large discrepancies exist, even at early times, for both FCFS and LAIN policies. A critical singularity at $t \approx 42$ forces the approximation to diverge strongly for the FCFS availability when $\eta = 0.40$.

We may conclude that a modification to the original model (4.4) and (4.5) is necessary to account for these difficulties. We do not attempt to investigate these modifications in this study.

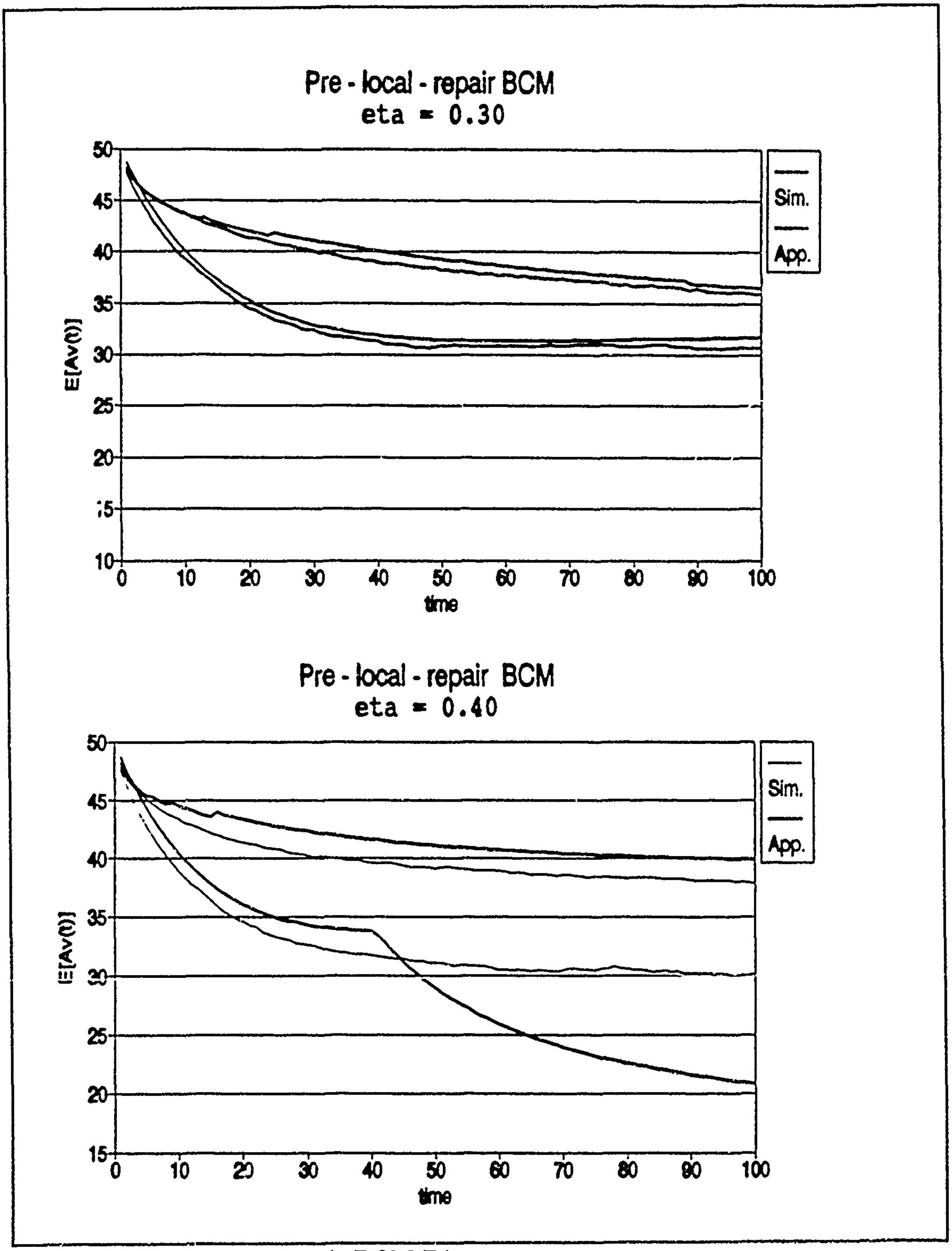

Figure B1. Pre-Local-Repair BCM Divergence

APPENDIX C. DIFFUSION APPROXIMATION ROUTINE

```
      SUBROUTINE DIFFUS (NTIME,MAXTIM,AVERAG,STDDEV,
     &                   DMEAN,DSTD,ITYPE)
      INTEGER QUANT
      PARAMETER (QUANT = 10)
      INTEGER NTIME,MAXTIM,ITYPE
      REAL AVERAG(NTIME),STDDEV(NTIME)
      REAL DMEAN(NTIME,QUANT),DSTD(NTIME,QUANT)
*
*     ........................ MODEL VARIABLES
      INTEGER ONREP,DOWN(QUANT),K(QUANT)
      INTEGER NUMMIN,AC,P,AV,TOTQU,AVMIN
      REAL LB(QUANT),NU(QUANT),W(QUANT)
      COMMON/MODEL1/ONREP,DOWN,K,NUMMIN,AC,
     &              P,AV,TOTQU,AVMIN
      COMMON/MODEL2/LB,NU,W
*
      INTEGER INOP(QUANT)
      COMMON/TIME0/INOP
*
*     ....................... POLICY VARIABLE
      INTEGER POLTYP
      COMMON/POLICY/POLTYP
*
*     ....................... DIFFUSION VARIABLES
      REAL MU(QUANT),ALPH(QUANT),Y0(QUANT),ACNORM,X0
      COMMON/APPROX/MU,ALPH,Y0,ACNORM,X0
*

C     .......................LOCAL VARIABLES
      INTEGER I,J,TSTEP
      REAL A


************************************************************************
*                    MAIN PROGRAM                                      *
************************************************************************

*     ....................... SCALING OF INPUT PARAMETERS
```

```
      A = REAL(AC)
      DO 10 I=1,QUANT
        A = A + REAL(K(I))
 10   CONTINUE
      ACNORM = REAL(AC) / A
      POLTYP = ITYPE
      DO 20 I=1,QUANT
        ALPH(I) = REAL(K(I)) / A
        MU(I) = NU(I) / A
        Y0(I) = REAL(INOP(I)) / A
 20   CONTINUE
*     ........................ COMPUTE TIME STEP
      TSTEP = MAXTIM/NTIME

*     ........................ CALL ODE SOLVER AND RETURN THE VALUES
*                              OF THE MEAN AND STANDARD DEVIATION FOR
*                              COMPONENT WITH LEAST AVAILABILITY AT
*                              EACH TIME, FOR TYPE "ITYPE"
      CALL DFMEAN(NTIME,MAXTIM,AVERAG,STDDEV,A,DMEAN,DSTD)
      RETURN
      END

************************************************************************
      SUBROUTINE DFMEAN(NTIME,MAXTIM,AVERAG,
     &                  STDDEV,A,DMEAN,DSTD)
************************************************************************

*     ...................... LOCAL ENVIRONMENT
      INTEGER QUANT,MXPARM,MXSTEP,METHOD
      REAL TOL,HINIT
      PARAMETER ( QUANT  = 10  )
      PARAMETER ( MXPARM = 50    )
      PARAMETER ( MXSTEP = 3000  )
      PARAMETER ( METHOD = 1     )
      PARAMETER ( TOL    = 1.0E-5)
      PARAMETER ( HINIT  = 1.0E-3)

*     ...................... INPUT/OUTPUT PARAMETERS
      INTEGER NTIME,MAXTIM
      REAL AVERAG(NTIME),STDDEV(NTIME),A
      REAL DMEAN(NTIME,QUANT),DSTD(NTIME,QUANT)

*     ........................ MODEL VARIABLES
```

```
      INTEGER ONREP,DOWN(QUANT),K(QUANT)
      INTEGER NUMMIN,AC,P,AV,TOTQU,AVMIN
      REAL LB(QUANT),NU(QUANT),W(QUANT)
      COMMON/MODEL1/ONREP,DOWN,K,NUMMIN,AC
     &          P,AV,TOTQU,AVMIN
      COMMON/MODEL2/LB,NU,W
*
*     ...................... DIFFUSION VARIABLES
      REAL MU(QUANT),ALPH(QUANT),Y0(QUANT),ACNORM,X0
      COMMON/APPROX/MU,ALPH,Y0,ACNORM,X0
*
*     ...................... LOCAL VARIABLES
      INTEGER IDO,ISTEP,I,J,INTIME,IMIN
      REAL AMAT(1,1),PARAM(MXPARM)
      REAL FCN,FCNJ
      REAL X,XEND,OK(QUANT),AVNOW,Y(QUANT+QUANT*QUANT)
*
      EXTERNAL FCN,FCNJ,IVPAG,SSET,ISMIN
*
*
*    y(1), y(2), ..., Y(quant) represent actual values for the scaled means.
*    Y(quant+1), y(quant+2),..., Y(quant+quant*quant) are a vector representation
*    of the variance-covariance matrix in  column form.
*
*     ...................... Set parameters for the ODE solver
      CALL SSET(MXPARM,0.0,PARAM,1)
      PARAM(1)  = HINIT
      PARAM(4)  = REAL(MXSTEP)
      PARAM(12) = REAL(METHOD)
*
      INTIME = MAXTIM / NTIME
*     ...................... Initialize queues and set initial time
      X = X0
      DO 10 I=1,QUANT
        Y(I) = Y0(I)
 10   CONTINUE
      DO 15 I=1,QUANT*QUANT
        Y(QUANT+I) = 0.0
 15   CONTINUE
      IDO=1
*     ...................... INTEGRATE SYSTEM OF ODE
      DO 200 ISTEP=NINT(X0)+INTIME,MAXTIM,INTIME
        XEND=REAL(ISTEP)
```

```
      CALL IVPAG(IDO,QUANT+QUANT*QUANT,FCN,FCNJ,AMAT,
   &           X,XEND,TOL,PARAM,Y)
      DO 100 I=1,QUANT
        IF(Y(I).LT.0.0) Y(I) = 0.0
        OK(I) = REAL(K(I)) - Y(I)*A
100   CONTINUE
*     ....................... FIND COMPONENT WITH LEAST AVAILABILITY
      IMIN = ISMIN(QUANT,OK,1)
*     ....................... FIND PRESENT AVAILABILITY
      AVNOW = REAL(AC)
      IF(OK(IMIN) .LT. AVNOW) AVNOW = OK(IMIN)
      AVERAG(ISTEP/INTIME) = AVNOW
*     ....................... COMPUTE STANDARD DEVIATION
      STDDEV(ISTEP/INTIME)=SQRT(ABS(A*Y(IPOS(IMIN,IMIN))))
      WRITE(11,111)ISTEP,(K(I)-Y(I)*A,I=1,QUANT)
      WRITE(11,111)ISTEP,(SQRT(ABS(A*Y(IPOS(I,I)))),I=1,QUANT)
111   FORMAT(1X,I3,10(1X,F5.1))
      DO 150 I=1,QUANT
          DMEAN(ISTEP/INTIME,I) = K(I) - Y(I)*A
          DSTD(ISTEP/INTIME,I)  = SQRT(ABS(A*Y(IPOS(I,I))))
150   CONTINUE
200 CONTINUE
*     ....................... RELEASE WORKSPACE
    IDO=3
    CALL IVPAG(IDO,QUANT+QUANT*QUANT,FCN,FCNJ,AMAT,
   &         X,XEND,TOL,PARAM,Y)
    RETURN
    END
*
************************************************************************
    SUBROUTINE FCN(N,X,Y,YPRIME)
************************************************************************
*
    INTEGER N
    REAL X,Y(N),YPRIME(N)
*
    INTEGER QUANT
    PARAMETER (QUANT = 10 )
*
*     ....................... MODEL VARIABLES
    INTEGER ONREP,DOWN(QUANT),K(QUANT),NUMMIN,AC
    INTEGER P,AV,TOTQU,AVMIN
    REAL LB(QUANT),NU(QUANT),W(QUANT)
```

```
      COMMON/MODEL1/ONREP,DOWN,K,NUMMIN,AC
     &              P,AV,TOTQU,AVMIN
      COMMON/MODEL2/LB,NU,W
*
*
*     ...................... DIFFUSION VARIABLES
      REAL MU(QUANT),ALPH(QUANT),Y0(QUANT),ACNORM,X0
      COMMON/APPROX/MU,ALPH,Y0,ACNORM,X0
*
*
*     ...................... POLICY VARIABLE
      INTEGER POLTYP
      COMMON/POLICY/POLTYP
*
*     ......................... LOCAL VARIABLES
      REAL PJ(QUANT),PSUM,AVNOW,Q(QUANT)
      REAL B(QUANT),H1(QUANT,QUANT),H(QUANT,QUANT)
      REAL SUMHS(QUANT),SUMPMU
      INTEGER IMIN,IPOS,I,J,L
*

*     ...................... Determine availability (AVNOW)
      IMIN = -1
      AVNOW = ACNORM
      DO 10 I=1,QUANT
        IF ((ALPH(I)-Y(I)) .LT. AVNOW) THEN
          AVNOW = ALPH(I)-Y(I)
          IMIN = I
        END IF
 10   CONTINUE
      PSUM=0.0
      DO 25 I=1,QUANT
*     ................. ... FCFS CASE
        IF(POLTYP .EQ. 2) THEN
          PJ(I) = Y(I)/MU(I)
*     ...................... LAIN CASE
        ELSE IF(POLTYP .EQ. 4) THEN
          PJ(I) = (W(I)/MU(I)) / (ALPH(I)-Y(I))**P
        ELSE
          WRITE(*,*)' ERROR IN POLICY TYPE'
          STOP
        END IF
        PSUM = PSUM + PJ(I)
```

```
 25  CONTINUE
*    ...................... COMPUTE ADAPTIVE PRIORITY (Q(i))
     DO 27 I=1,QUANT
       IF(PSUM .NE. 0.0) THEN
         Q(I) = PJ(I) / PSUM
       ELSE
         Q(I) = 1.0 / REAL(QUANT)
       END IF
 27  CONTINUE
*    ...................... Compute scaled means
     DO 30 J=1,QUANT
       YPRIME(J)=LB(J) * AVNOW - (MU(J) * Q(J))
 30  CONTINUE
*    ...................... Compute scaled variance-covariance
     SUMPMU = 0.0
     DO 50 I=1,QUANT
       SUMPMU = SUMPMU + (Q(I)/MU(I))
 50  CONTINUE
     DO 200 I=1,QUANT
       B(I) = LB(I)*AVNOW+MU(I)*Q(I)*(1.0+2.0*Q(I)
    &          *(MU(I)*SUMPMU-1.0))
       IF(POLTYP .EQ. 2) THEN
*    ...................... FCFS
           IF(Y(I) .NE. 0.0) THEN
               H1(I,I) = (-MU(I)/Y(I)) * (Q(I)*(1.0-Q(I)))
           ELSE
               H1(I,I) = (-1.0) * (Q(I)*(1.0-Q(I)))
           END IF
       ELSE
*    ...................... LAIN CASE
           H1(I,I) = (-MU(I)*REAL(P)/(ALPH(I)-Y(I)))
    &                *(Q(I)*(1.0-Q(I)))
       END IF
       DO 100 J=1,QUANT
         IF(J .NE. I) THEN
           IF(POLTYP .EQ. 2) THEN
*    ...................... FCFS CASE
               IF(Y(I) .NE. 0.0) THEN
                   H1(I,J) = (MU(I)/Y(I))* Q(I)*Q(J)
               ELSE
                   H1(I,J) = Q(I) * Q(J)
               END IF
           ELSE
```

```
*     ........................ LST AV
                H1(I,J) = (MU(I)*REAL(P)/(ALPH(I)-Y(I)))
     &                     * Q(I) * Q(J)
            END IF
          END IF
100     CONTINUE
200   CONTINUE
      DO 400 I=1,QUANT
        SUMHS(I) = 0.0
        DO 300 J=1,QUANT
          IF((IMIN .GT. 0) .AND. (J .EQ. IMIN)) THEN
            H(I,J) = H1(I,J) - LB(I)
          ELSE
            H(I,J) = H1(I,J)
          END IF
          SUMHS(I) = SUMHS(I) + H(I,J)*Y(IPOS(I,J))
300     CONTINUE
400   CONTINUE
      DO 600 I=1,QUANT
        YPRIME(IPOS(I,I)) = B(I) + 2.0 * SUMHS(I)
        DO 500 J=1,QUANT
          IF(J .NE. I) THEN
            YPRIME(IPOS(I,J)) = 0.0
            DO 450 L=1,QUANT
              YPRIME(IPOS(I,J)) = YPRIME(IPOS(I,J)) +
     &          H(I,L)*Y(IPOS(J,L)) + H(J,L)*Y(IPOS(I,L))
450         CONTINUE
          END IF
500     CONTINUE
600   CONTINUE
      DO 700 I=1,QUANT
        IF(Y(I) .LT. 0.0) Y(I) = 0.0
700   CONTINUE
      RETURN
      END
**********************************************************************
      INTEGER FUNCTION IPOS(I,J)
**********************************************************************
*
*  This function maps matrix-like indices to the vector locations in Y
*
      INTEGER I,J,QUANT
      PARAMETER (QUANT = 10 )
```

```
IPOS = QUANT + I + (J-1) * QUANT
RETURN
END
```

APPENDIX D. HEAVY TRAFFIC CONDITION

In this appendix we show that if the *heavy traffic conditions* in (1.2) do not hold, the diffusion approximation model is profoundly degraded. Consider the sample system described in Table 1D. Applying (1.2) to the data yields

$$50 \times \sum_{i=1}^{10} \frac{\lambda_i}{\nu_i} = 0.85 < 1 .$$

Table 1D INPUT DATA

MODULE	K_i	λ_i	ν_i
1	50	0.005	5.0
2	50	0.004	5.0
3	50	0.003	5.0
4	50	0.002	5.0
5	50	0.001	5.0
6	50	0.009	2.5
7	50	0.008	2.5
8	50	0.007	2.5
9	50	0.006	2.5
10	50	0.005	2.5

The heavy traffic condition is obviously violated and we want to verify how this situation will influence the accuracy of the diffusion model. Figure 1D displays the expected value of the combat availability for t between 0 and 100. We can see that the analytical results show a wide variability when compared to the simulation.

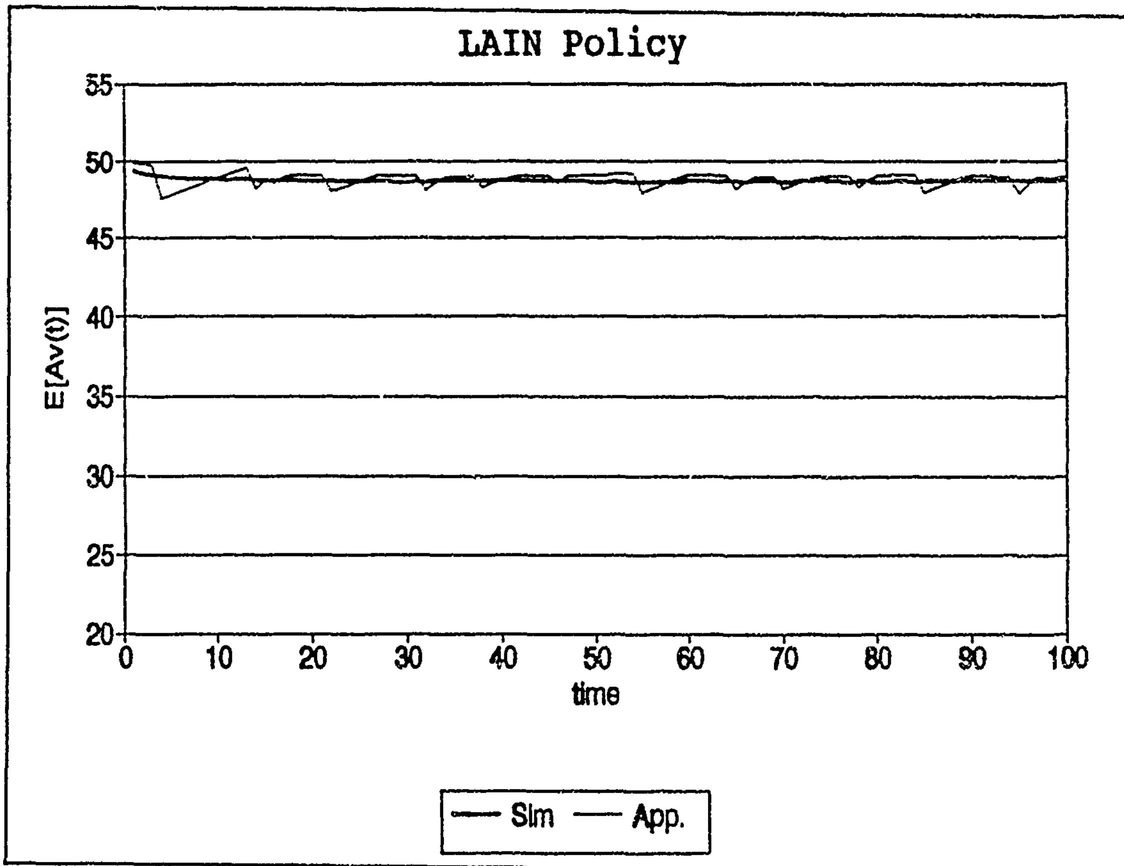

Figure 1D. LAIN Policy (Mean)

Figure 2D shows that the performance with respect to the calculation of variances is extremely poor.

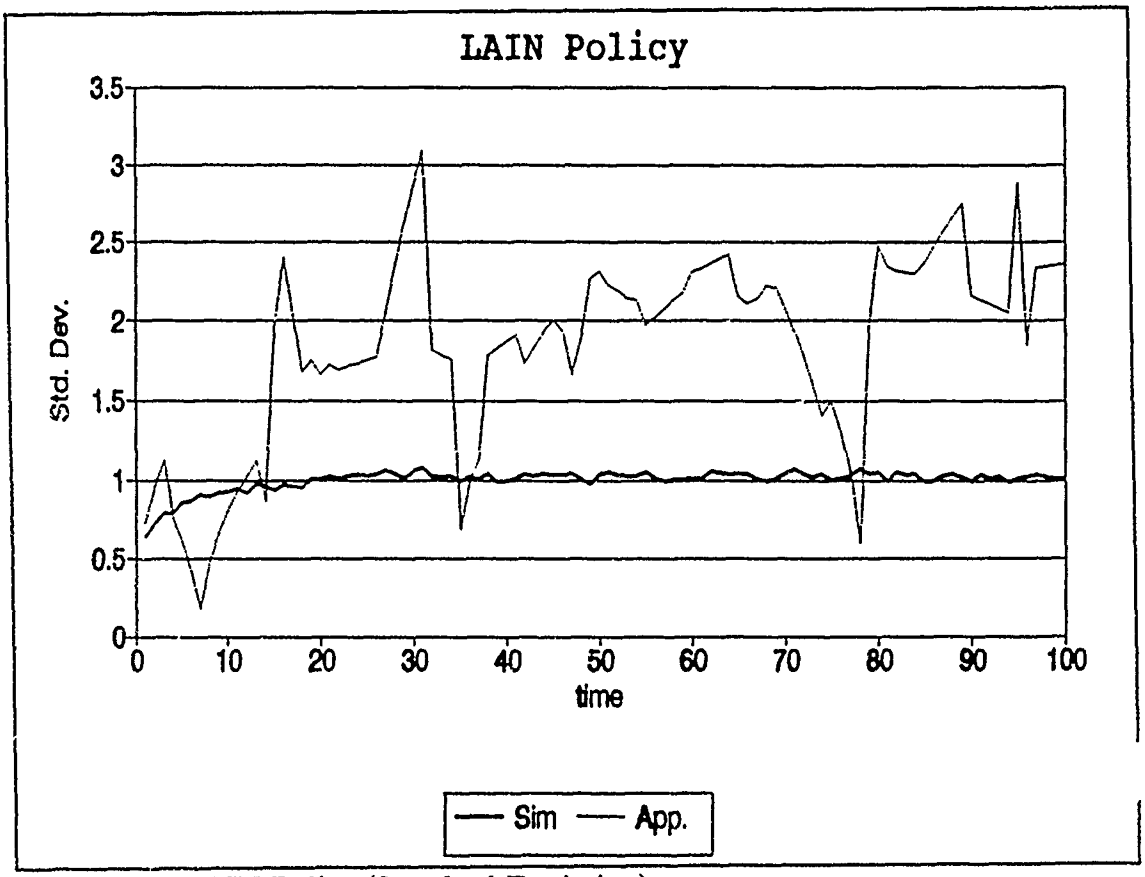

Figure 2D. LAIN Policy (Standard Deviation)

LIST OF REFERENCES

1. Latta, Peter J., *A Comparison of Six Repair Scheduling Policies for the P-3 Aircraft*, Master's Thesis, Naval Postgraduate School, Monterey, California, 1988.

2. Karlin, S., and Taylor, H. M., *A Second Course in Stochastic Processes*, Academic Press, 1981, Chapter 15.

3. Arnold, L., *Stochastic Differential Equations: Theory and Applications*, Wiley & Sons, 1974

4. Gaver, D. P., and Jacobs, P. A., "Processor-Shared Time-Sharing Models in Heavy Traffic", *SIAM Journal of Computing*, v. 15, pp. 1085-1100, November 1986.

5. Gaver, D. P., and Lehoczky, J. P., "Gaussian Approximations to Service Problems: a Communication System Example", *Journal of Applied Probability*, v. 13, pp. 768-780, 1976.

6. Gaver, D. P., and Lehoczky, J. P., *A Diffusion Approximation for a Repairman Problem with Two Types of Repair,* Naval Postgraduate School Technical Report, NPS-55-77-3, January 1977.

7. Iglehart, D. L., "Limiting Diffusion Approximations for the Man[illegible]er Queue and Repairman Problem", *Journal of Applied Probability*, v. 2, pp [illegible]41, 1965.

8. Pilnick, Steven E., *Combat Logistic Problems,* Ph.D. Dissertation, Naval Postgraduate School, Monterey, California, 1989.

9. Gaver, D. P., Isaacson, K., and Pilnick S. E., *Assessing and Improving Many-Platform Availability During Intensive Demand Periods*, To be Published, 1990.

10. Lewis, P. A., and Uribe, L., *The New Naval Postgraduate School Random Number Package - LLRANDOMII*, Naval Postgraduate School Technical Report, NPS-55-81-005, 1981.

11. IMSL release 10, *Math/Library User's Manual*, IMSL, 1987.

12. Lewis, P. A. W., and Orav, E. J., *Simulation Methodology for Statisticians, Operations Analysts, and Engineers*, Wadsworth & Brooks/Cole, 1989.

13. PC-MATLAB, *MATLAB User's Guide*, The MathWorks, Inc., 1988.

14. Wets, R. J.-B., "Stochastic Programming", *Handbooks in Operations Research and Management Science, Vol 1, Optimization*, Chapter VIII, North-Holland, 1989.

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